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Beyond the Arrow effect: a Schumpeterian theory of multi-quality firms *

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Abstract

This paper introduces multi-quality firms within a Schumpeterian framework. Featuring non-homothetic preferences and income disparities in an otherwise standard quality-ladder model, we show that the resulting differences in the willingness to pay for quality among consumers generate both positive investments in R&D by industry leaders *and* positive market shares for more than one quality, hence allowing for the emergence of multi-product firms within a vertical innovation framework. This positive investment in R&D by incumbents is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers: in our framework, the incentive for a leader to invest in R&D stems from the possibility for an incumbent having innovated twice in a row to efficiently discriminate between rich and poor consumers displaying differences in their willingness to pay for quality. We hence exemplify a so far overlooked demand-driven rationale for innovation by incumbents. Such a framework also makes it possible to analyze the impact of inequality both on long-term growth and on the allocation of R&D activities between challengers and incumbents. We find that an increase in the income *gap* positively impacts an economy's growth rate, partly shifting R&D activities from challengers to incumbents. On the other hand, a greater income *concentration* is detrimental for growth, diminishing both the incumbents' and the challengers' R&D activities.

Keywords: Growth, Innovation, Income inequality, Multi-Product firms.

JEL classification: O3, O4, F4.

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1 Introduction

The importance and specificities of multi-product firms (MPFs) have lately been exemplified by a growing body of literature.[1] In particular, because of unique supply and demand linkages, MPFs' product-market decisions such as intra-firm portfolio adjustments or investment in product innovation have been shown to obey to specific incentives (Eckel and Neary, 2010; Dhingra, 2013). Dynamic R&D-driven growth models studying the behavior and impact on aggregate innovation of MPFs have already been provided for the cases where firms are multi-industry (Klette and Kortum, 2004; Akcigit and Kerr, 2010) or multi-varieties (Minniti, 2006). However, the standard quality-ladder framework has so far not been able to account for the existence of "multi-quality" firms, i.e. firms selling more than one quality-differentiated version of the same good. Indeed, the "creative destruction" mechanism at the heart of Schumpeterian models traditionally not only deters leaders from investing in R&D, but *also* guarantees the systematic exit of any quality that has moved away from the frontier.[2]

Examples of firms offering more than one quality-differentiated version of the same product however abound. Apple recently jointly launched its latest flagship phone, the iPhone 6s, along with two lower-cost versions (iPhone 6 and iPhone 5S); the iPad is now declined in iPad pro, iPad Air and iPad mini. Similarly, Intel commercializes a whole array of microchips, selling its latest, highly efficient processors at high prices (Xeon, Core) while simultaneously offering cheaper models further from the industry frontier (Celeron, Atom). In the car industry, Renault-Nissan launched in the last decade several low-cost cars specifically marketed for developing countries (Dacia's Logan initially designed for the Eastern European markets, Datsun's Go programmed to be launched in 2017 on the Indian market), re-using obsolete technologies previously featured in leading brands of the constructor (Renault Clio for the Logan, Nissan Micra for the Go). Those examples show how firms resort to *vertical* brand diversification so as to give a second life to technologies having moved down the quality ladder, and how they are motivated to do so by income inequality, i.e. so as to better price-discriminate among consumers having different purchasing powers.

The present paper builds on this body of anecdotal evidence, and provides a model accounting for the existence of **multi-quality leaders** within a dynamic Schumpeterian framework. More precisely, along the salient features of the examples described above, we argue that as long as we allow preferences to be non-homothetic, income distribution impacts the strength and scope of the "creative destruction" process. Income differences

[1] Among others, Bernard et al. (2010) estimate that MPFs account for 41% of the total number of US firms as well as for 91% of total output; also, they estimate that the contribution to the US output growth of product mix decisions of MPFs (i.e. product adding and dropping) is greater than the one of firm entry and exit.

[2] Mussa and Rosen (1978) study pricing decisions of multi-quality firms, but in a static framework precluding any specific modeling of the R&D process leading to the initial design of the product line. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) feature MPFs in a quality-ladder world; however, multi-product firms are also multi-industry firms in their models, with only one quality being sold within each product line.

then account for both the survival of more than one quality at the equilibrium *and* for positive investment in R&D by incumbents. The result is the endogenous emergence in a dynamic framework of multi-quality leaders whose product portfolio composition and investment in R&D activities are both influenced by the extent of income disparities.

The intuition behind this result is straightforward, and is related to the well-explored notion of second-degree price discrimination. For a monopolist, serving costumers who do not care much for quality creates negative externalities, since it hinders the captation of costumer surplus from those who have a stronger taste for quality. Mussa and Rosen (1978) have demonstrated that a monopolist confronted to such disparities in consumers' taste for quality optimally chooses to offer lower quality items charged at a lower price to the less enthusiastic consumers, opening the possibility of charging higher prices to more adamant buyers of high quality units. In their microeconomic *static* set-up, the monopolist has by assumption a whole product line at its disposition. In a standard quality-ladder dynamic framework on the other hand, the monopolist only has access to as many qualities as times he has innovated. We demonstrate that in such a dynamic set-up, internalization of such negative externalities then leads to investment in R&D by incumbent monopolists, and in case of success, to the existence of firms simultaneously offering more than one quality of their product.

We first demonstrate the general nature of the identified price-discrimination mechanism in a partial equilibrium framework. We show that provided there exists differences in the willingness to pay for quality among consumers, the expected value of innovating once more differs between challengers and incumbents: the Arrow effect operating under free entry then becomes compatible with positive investment in R&D by incumbents. We then integrate such a mechanism in a Schumpeterian model by featuring non-homothetic preferences in an otherwise traditional quality-ladder framework, hence allowing for several different qualities to be consumed at the equilibrium in the presence of differences in wealth endowment.[3] In such a framework, a *challenger* winning the latest innovation race and being the producer of the highest quality needs to decide between two alternatives: capturing the whole market by charging a price sufficiently low to appeal to the poorest households, *or* selling its product at a higher price only to the wealthiest consumers, at the cost of abandoning the rest of the market to its direct competitor (i.e. the previous quality leader). On the other hand, an *incumbent* winning an innovation race retains exclusive monopoly rights over two successive qualities: he can then efficiently discriminate between rich and poor consumers by offering two distinct price/quality bundles, capturing the whole market *and* reaping the maximum surplus from the wealthy consumers at the same time.

[3]This property is obtained by imposing unit consumption of quality goods in a two-class society, the rest of a consumer's income being spent on standardized goods: within each industry, a given consumer then buys the quality that, *given its price*, offers him the highest utility. By contrast, in the standard quality-ladder models (Segerstrom et al., 1990; Grossman and Helpman, 1991a; Aghion and Howitt, 1992), the quality goods are divisible, the quality goods are *divisible*, ensuring that only the highest quality is consumed at the equilibrium, even in the case of wealth endowment disparities: the poorest consumers only consume a lower amount of the top quality good.

We then model R&D races in which both incumbents and challengers are participating, and show that without any advantage of any kind in the R&D field and under free entry, the incumbent still invests a strictly positive amount in R&D. Such a behavior directly stems from the existing increment between the profits realized when being a successful challenger and a successful incumbent.

We then finally move to studying the impact of income distribution on the innovation incentives of both challengers and incumbents, and by extension on long-term growth. We show that in a quality-ladder model, the impact on growth of an increase in the inequality level depends on the nature of the considered shock. More precisely, an increase in the income *gap* has a positive impact on the R&D activities of both incumbents an challengers, hence increasing an economy's growth rate. On the other hand, a greater income *concentration* is unequivocally detrimental for growth, diminishing both the incumbents' and the challengers' R&D investments. Indeed, in that case the positive price effect stemming from a wealthier rich class is systematically more than offset by the negative market size effect resulting from the decrease in the number of rich consumers.

The main contribution of this paper is to provide a dynamic framework endogenously accounting for the emergence of multi-quality leaders in the presence of income disparities among consumers. Beyond its novelty, such a result bears several implications. First, while so far the incentives for innovation by quality leaders have essentially been modeled as stemming from the structure of the R&D process (i.e. from the *supply* side), this paper is the first to provide a *demand-driven* incentive for investment in R&D by incumbents. Second, such a framework makes it possible to investigate the impact of income distribution on the intensity of both challengers' *and* incumbents' innovation activities, a feature that totally overturns the predictions that had so far been obtained in the quality-ladder literature regarding the interactions of growth and inequality operating through the product market (Zweimuller and Brunner, 2005).

Relation to literature.

This paper contributes to the literature accounting for innovation by incumbents in quality-ladder models. Segerstrom and Zolnierek (1999) as well as Segerstrom (2007) have obtained positive incumbent investment in R&D by assuming that the expertise granted by quality leadership confers R&D cost advantages. Etro (2004, 2008) models sequential patent races with concave R&D costs where the incumbent, acting as a Stackelberg leader, is given the opportunity to make a strategic precommitment to a given level of R&D investment: the quality leader then has an incentive to invest in R&D in order to deter outsiders' entry. Denicolo and Zanchettin (2012) as well as Acemoglu and Cao (2015) provide models where incumbents and challengers participate to two different kinds of R&D races, differing in terms of costs and rewards: leaders invest in R&D to improve their products (incremental innovation), while challengers participate to R&D races in the hope of leapfrogging the existing incumbent (radical innovation). All those models have hence explored various possible incentives for innovation by incumbent stemming from the structure of the R&D process, i.e. from the *supply side*. While all those channels are indeed certainly relevant, this paper explores another venue and provides a *demand-based* rationale for leader R&D, stemming from the perspective of more efficient price discrimination in the case of successive successful innovations. All those papers also feature homothetic preferences, hence guaranteeing that even in the presence of consumer heterogeneity, only the highest quality will be produced and consumed within each industry: the emergence of multi-quality leaders cannot be a consequence of positive innovation by incumbent in those models.

A paper more closely related to this work is the one of Aghion et al. (2001), who analyze the influence of product market competition on innovation intensity, developing a framework in which goods of different quality are imperfect substitutes and can therefore coexist in the market. They show that the perspective to lessen the competition pressure (and broaden the market share) provides the incentive for the incumbent to resort to step-by-step innovation in order to improve its own product. They however preclude free entry by exogenously imposing that only two firms are active and invest in R&D, while our paper on the other hand provides a product market-driven incentive that is robust to the free entry condition.

This work also contributes to the small literature studying the R&D investment of multi-product firms in a dynamic, general equilibrium framework. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) have already provided quality-ladder models in which industry leaders invest in exploration R&D so as to expand their activities in *other* sectors; those frameworks however cannot account for leaders widening their product portfolio within a *given* industry. Minniti (2006) embeds multi-product firms selling more than one horizontally-differentiated variety of a given good in an endogenous growth model; however, his model is an expanding-variety one, hence precluding the emergence of multi-*quality* firms.

This paper is finally also related to the literature examining the relationship between long-term growth and income distribution operating through the demand side. Foellmi and Zweimuller (2006) demonstrate that in an expanding-variety framework, higher inequality levels are systematically beneficial for long-term growth. Foellmi et al. (2014) provide a model combining both product innovations (introducing new luxury goods) and process innovations (transforming those goods into necessities through mass production technologies): in such a framework, the impact of higher inequality is ambiguous on growth, and depends on the scope of the productivity gains stemming from the process innovations. Both those contributions however investigate the impact of income distribution on growth in a *horizontal* differentiation framework, where firms retain *permanent* monopoly rights over their *single* product. Li (2003) and Zweimuller and Brunner (2005) on the other hand have studied the impact of disparities in purchasing power of households in a quality-ladder framework. Zweimuller and Brunner (2005) in particular show that a reduction in the level of inequality through a reduction in the income gap is beneficial for innovation intensity and hence for growth. They however only consider the R&D investment of challengers, and overlook the existing incentives for incumbent innovation in the presence of differences in the willingness to pay of consumers. We demonstrate that taking into account the R&D investment by incumbents actually totally changes the predictions regarding the overall growth rate of the economy: it reverses them in the case of an increased income gap, and enables to study the impact of an increased income concentration.

The rest of the paper is organized as follows. Section 2 illustrates in a simple partial equilibrium framework how differences in the willingness to pay for quality impact the innovation incentives of both challengers and incumbents. Section 3 presents the structure of our general equilibrium model, while section 4 studies its steady state properties. Section 5 then analyzes the effects of the extent of inequality on the innovation intensity. Section 6 concludes.

2 Reconciling the Arrow effect with incumbent's innovation

In order to demonstrate the generality of the mechanism driving the emergence of multi-quality leaders in our model, we first isolate it within a partial equilibrium framework. we hence model R&D races meeting the most standard assumptions of the baseline Schumpeterian growth model (Barro and i Martin (2003), chapter 7; Acemoglu (2008), chapter 14).

More precisely, we consider the R&D investment decisions of firms aiming at entering a final good industry characterized by an array of quality-differentiated products. Each innovation increases the quality by a rung q, with the n-th innovation being of quality q^n. The successful researcher retaining the exclusive rights over the latest technology obtains a flow of monopoly profits $\pi(n)$.[4] We assume that the probability to innovate $p(n)$ in an industry where the highest quality currently available is q^n depends linearly on the total expenditures over R&D $\phi(n)$: more precisely, we have $p(n) = Q(n)\phi(n)$, with $Q(n)$ capturing the effect of the current technology position n.[5] The expected value of an innovation is then $E[v(n)] = \frac{\pi(n)}{r+p(n)}$, with r being the interest rate over time (we consider the steady state of such an economy, and hence assume r to be constant). We assume that both challengers and incumbents have the *possibility* to invest in R&D, and denote by $\phi_C(n)$ and $\phi_I(n)$ the respective amounts being invested.

In an industry where the highest quality currently available is q^n, the standard, free-entry condition for challengers equates the costs incurred when engaging in R&D $\phi_C(n)$ and the expected value of innovating $p(n)E[v(n+1)]$:

$$\phi_C(n)\left(1 - Q(n)E[v(n+1)]\right) = 0 \tag{1}$$

[4]Indeed, whether he needs to resort to limit pricing or can charge the unconstrained monopoly price, the successful innovator is systematically able to charge a price that will ensure him a monopoly position (Grossman and Helpman, 1991b; Aghion and Howitt, 1992).

[5]We hence do not impose decreasing returns, neither at the firm nor at the industry level.

On the other hand, the Hamilton-Jacobi-Bellman equation of the incumbent deciding whether to invest in R&D or not is of the form:

$$rv(n) = \max_{\phi_I(n)\geq 0}\{\pi(n) - \phi_I(n) + Q(n)\phi_I(n)(E[v(n+1)] - E[v(n)]) - Q(n)\phi_C(n)E[v(n)]\}$$

with the first order condition (f.o.c.) being:

$$(\underbrace{-1 + Q(n)E[v(n+1)]}_{(*)} \underbrace{-Q(n)E[v(n)]}_{(**)})\phi_I(n) = 0$$

The value of the $(*)$ term is null under the free-entry condition (1). The remaining term $(**)$ is negative, and represents the well-known "Arrow effect", capturing the fact that the incumbent would loose its current profits if it innovated a second time. We are hence confronted to the classic result that under free-entry, incumbents do not have any incentive to carry out research in a vertical framework, since they would cannibalize their own market in case of a successful innovation.[6]

This result relies on the "creative destruction" phenomenon at work in quality-ladder models: since a new quality has an objective advantage over all the previous ones, its producer can (and will) exclude all the other competitors from the market. However, the industrial organization literature studying competition and pricing decisions in vertically-differentiated markets has since long shown that quality differentiation *does not* preclude the survival of more than one quality and/or more than one producer. Indeed, provided there exist *differences in the willingness to pay for quality among consumers*, strategic pricing of firms in a situation of natural oligopoly or monopoly will lead to more than one quality being sold and consumed at the equilibrium. Such differences among consumers in the price they are ready to pay for a given quality are generated either by income differences among consumers displaying non-homothetic preferences[7] (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982), or by exogenously imposed different tastes for quality (Mussa and Rosen, 1978; Glass, 1997). In such a framework, Gabszewicz and Thisse (1980) as well as Shaked and Sutton (1982) have shown that competition among vertically-differentiated firms yields several qualities being sold at different prices at the equilibrium (the total number of qualities is however naturally limited by the existence of marginal production costs increasing along quality). Similarly, Mussa and Rosen (1978) have proved that a monopoly firm having at its disposal a whole product line and being unable to perfectly discriminate among heterogenous consumers[8] offers a whole menu comprising different qualities sold at

[6] As already stated in our literature review, models where incumbents innovate have already been provided (Aghion et al., 2001; Segerstrom, 2007; Etro, 2008; Acemoglu and Cao, 2010). However, they all depart in one way or the other from the standard specification I outlined in my example.

[7] Indeed, income differences alone do not guarantee differences in the willingness to pay: in the case of homothetic preferences such as the standard quality-augmented CES utility function, the constant elasticity of substitution along income will lead poor and rich individuals to consume the same quality, but in different amounts.

[8] Perfect discrimination means that a monopolist can distinguish among consumers prior to any actual sale, and charge different prices to different consumers for the same good.

different prices. To sum it up, "*vertical product differentiation refers to a class of products which cohabit simultaneously on a given market, even though customers agree on a unanimous ranking between them. (...)The survival of a low-quality product then rests on the seller's ability to sell it at a reduced price, (...) specializing in the segment of costumers whose propensity to spend is low, either because they have relatively lower income, or relatively less intensive preferences, than other costumers*" (Gabszewicz and Thisse, 1986).

We hence claim that *provided consumers display differences in their willingness to pay for quality*, the profits realized by a firm having a product line comprising two qualities are *superior* to the profits realized by a firm having the knowledge to produce only one quality level. Indeed, a firm being able to produce and sell two qualities will be able to better *discriminate* among consumers differing in their willingness to pay, capturing the incremental profits generated by charging a higher price to quality-loving consumers, while still offering a lower quality (charged at a lower price) to consumers less prone to value quality. In other words, we claim that in a framework allowing for differences in the willingness to pay to arise, the expected value of being the winner of the next innovation race is higher for the incumbent than for the challenger: $E[v_I(n+1)] > E[v_C(n+1)]$. Taking into account those different valuations of further innovating, the free-entry condition for challengers then becomes:

$$\phi_C(n)\left(1 - Q(n)E[v_C(n+1)]\right) = 0 \tag{2}$$

while the HJB equation of the incumbent yields the following f.o.c.:

$$(\underbrace{-1 + Q(n)E[v_I(n+1)]}_{>0}\underbrace{-Q(n)E[v_C(n)]}_{(**)})\phi_I(n) = 0$$

The negative cannibalization term $(**)$ is now compensated by a positive term. **The Arrow effect is hence a priori not incompatible with investment in R&D by incumbents any more.** Indeed, as long as the incumbent has not fully exploited the price discrimination possibilities offered when having more than one quality at one's disposal, the free entry condition will *not* preclude a positive amount being invested in R&D by incumbents.

Having precisely identified the mechanism at work in a partial equilibrium framework, we now present an economy displaying the required feature, i.e. differences in the willingness to pay for quality among consumers. More precisely, we model non-homothetic preferences through unit consumption of the quality good, and incorporate this feature in an otherwise canonical quality-ladder framework displaying income inequality.

3 The model

It will first prove useful to provide a quick sketch of the model architecture. We consider an economy featuring a continuum of industries producing quality-differentiated goods; heterogenous consumers in terms of wealth consume both those goods and a composite standardized good. Firms operating in the differentiated industries proceed to R&D so as to improve the quality of the final consumption goods; they decide on the amount invested in R&D by taking into account that once they have successfully innovated, the existing disparities in the consumers' income will have an impact on their pricing strategy and the corresponding profits. Labor is divided between the production of the quality-differentiated goods, the production of the aggregate homogenous good, and participation to the R&D sector. In such a *product*-innovation model (i.e. without any mechanism ensuring productivity improvements), the balanced growth path is characterized by constant levels of innovation, overall wealth and consumption; consumers however still become better-off over time due to the quality improvements of the differentiated goods and the resulting growth of individual utility. We now move to presenting the details of the model.

3.1 Consumers

The economy is populated by a fixed number L of consumers that live infinitely and supply one unit of labor each period, paid at a constant wage w. While all consumers are identical with respect to their preferences and their labor income, they are assumed to differ in terms of wealth, based on firms' assets ownership. More precisely, we assume a two-class society with rich (R) and poor (P) consumers being distinguished by their wealth $\omega_R(t)$ and $\omega_P(t)$.[9]

The share of "poor" consumers within the population is denoted by β. The extent of inequality within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate stock of assets $\Omega(t)$. $d \in (0,1)$ is defined as the ratio of the value of the stock of assets owned by a poor consumer *relative* to the average per-capita wealth: $d = \frac{\omega_P(t)}{\Omega(t)/L}$. The wealth position of the rich can be computed for a given d and β, and we finally have $\omega_P(t) = d\frac{\Omega(t)}{L}$ and $\omega_R(t) = \frac{1-\beta d}{1-\beta}\frac{\Omega(t)}{L}$.

Current income $y_i(t)$ of an individual belonging to the group i ($i = P, R$) is then of the form:

$$y_i(t) = w + r(t)\omega_i(t) \tag{3}$$

with $r(t)$ being the interest rate.

The existence of such income disparities among consumers is however not sufficient to generate variations in the quality choice along income. Indeed, in the case of standard quality-ladder models traditionally featuring quality-augmented CES utility functions, the

[9] All the results presented in the paper pertaining to investment in R&D by incumbents are robust under the alternative specification of inequality being generated through differences in income, i.e. through different endowments in labor efficiency units.

homotheticity of the preference specification guarantees that both poor and rich consumers end up purchasing the **same** quality, but in **different amounts**. So as to obtain the possibility of different quality choices for different consumer groups, I hence consider **non-homothetic preferences** in the form of a **unit consumption** requirement (i.e. the consumption of a given quality good yields a positive utility only for the first unit, and zero utility for any additional unit).[10]

More precisely, two types of final goods are available within the economy. One group of products, indexed by $s \in [0, 1]$, is subject to quality innovation over time. At any date t, the following "quality ladder" determines the current highest quality $q_n(s, t)$ available within one industry:

$$q_n(s, t) = k^{n(s,t)} q_0(s, 0)$$

with $k > 1$, and where $n(s, t)$ denotes the number of innovations in industry s between time 0 and t. For the sake of simplicity, we also set $q_0(s, 0) = 1 \; \forall \; s \in [0, 1]$. Two successive quality levels j and $j - 1$ hence differ by a fixed factor $k > 1$: $q_j(s, t) = k.q_{j-1}(s, t)$. As stated above, the crucial assumption is that quality goods are **indivisible**, and consumers value at most **one unit** of each differentiated good. For each industry s at each period t, an individual belonging to group i then needs to proceed to two distinct choices:

(1) whether to consume or not good s;

(2) *if* he does choose to consume good s, he then needs to determine what quality level $q_j(s, t)$ (with $j \in [0, n(s, t)]$) offers him the highest utility *given its price* $p_j(s, t)$.

Considering these assumptions, let $x_i(s, t)$ be an indicator function that takes value 1 if a consumer belonging to group i consumes good s at time t (and 0 otherwise), $q^i_j(s, t)$ be the chosen quality by a type i consumer in sector s at time t, and $Q_i(t) = \int_0^1 x_i(s, t) q^i_j(s, t) ds$ the (group-specific) index of consumed qualities over industries.

Consumers then spend the rest of their income over the consumption of $c_i(t)$ units of a composite standardized commodity. This homogenous good is produced with a unit labor input of $1/w$; being competitively priced, it hence serves as the numeraire.

At time τ, the objective function of a type i consumer is given by:

$$\mathcal{U}_i(t) = \int_\tau^\infty \ln(c_i(t) Q_i(t)) e^{-\rho(t-\tau)} dt \tag{4}$$

with ρ being the rate of time preference. Households make consumption choices both within and across periods so as to maximize the above lifetime utility subject to the lifetime budget constraint

$$\int_\tau^\infty (c_i(t) + P_i(t)) e^{-R(t,\tau)} dt \leq \omega_i(\tau) + \int_\tau^\infty w e^{-R(t,\tau)} dt \tag{5}$$

[10]Unit consumption of the quality-differentiated goods ensures the non-homotheticity of the preference structure in this model. This particular way to model non-homotheticity is the most classic in qualitative choice models featuring strategic pricing of firms (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982). One could also have obtained differences in the willingness to pay by imposing exogenously different tastes for quality (Glass, 1997).

where $R(t,\tau) = \int_\tau^t r(s)ds$ is the cumulative discount factor between times τ and t, $r(t)$ is the interest rate at time t, $\omega_i(\tau)$ is the initial wealth level held by a type i consumer, and $P_i(t) = \int_0^1 x_i(s,t)p_j(s,t)ds$ is the (group-specific) price index associated to the quality good consumption index $Q_i(t)$.

We first characterize the intra-temporal consumption choices of the consumers. The first-order condition associated to the consumption of the homogenous good $c_i(t)$ is:

$$\frac{1}{c_i(t)} = \lambda_i(t) \tag{6}$$

with $\lambda_i(t)$ being the marginal utility of wealth at time t (i.e. the current-value multiplier). On the other hand, the first-order conditions for the **discrete consumption choice** of the quality-differentiated goods are of the form:

$$\begin{aligned} &\{x_i(s,t), q_j^i(s,t)\} \\ &= \begin{cases} \{1, k^{n(s,t)}\} & \text{if } \mu_i(t)k^{n(s,t)} - p_n(s,t) \geq \max\left[\mu_i(t)k^{n(s,t)-1} - p_{n-1}(s,t), ..., \mu_i(t) - p_0(s,t), 0\right] \\ \{1, k^{n(s,t)-1}\} & \text{if } \mu_i(t)k^{n(s,t)-1} - p_{n-1}(s,t) \geq \max\left[\mu_i(t)k^{n(s,t)} - p_n(s,t), ..., \mu_i(t) - p_0(s,t), 0\right] \\ ... & \\ \{1, 1\} & \text{if } \mu_i(t) - p_0(s,t) \geq \max\left[\mu_i(t)k^{n(s,t)} - p_n(s,t), ..., \mu_i(t)k - p_1(s,t), 0\right] \\ \{0, .\} & \text{otherwise} \end{cases} \end{aligned} \tag{7}$$

with $\mu_i(t) = \frac{1}{\lambda_i(t)Q_i(t)}$ being household's i willingness to pay per unit of quality. Those f.o.c.s state that: (1) for a type i consumer, the purchase of a unit of the quality good s needs to represent a positive utility gain; (2) provided a utility gain is possible, good s will be purchased at the quality level that offers the highest gain *given* its price and the price of every other available quality within industry s.[11]

We now move to the inter-temporal aspect of the consumer problem. First, separability of utility (both over time and across goods) guarantees that for any given foreseen time path $P_i(t)$ of expenditures devoted to the continuum of quality goods, the optimal time path of consumption expenditures $c_i(t)$ on homogenous commodities has to fulfill the standard first-order condition of such an intertemporal maximization problem:

$$\frac{\dot{c}_i(t)}{c_i(t)} = r(t) - \rho \tag{8}$$

The time path of $Q_i(t)$, on the other hand, is the combined result of (i) the successive stochastic quality jumps occurring in every quality-differentiated industry s, and (ii) the resulting optimal consumer choices *given* the prices charged for the different qualities available within industry s. We will hence go back to fully characterizing it once we have described the firms' pricing and R&D decisions (see equation (16) in section 3.2.3) and established the BGP properties of such a model (see Proposition 1 and footnote 19 in section 3.2.4). It is already worth mentioning that in *product*-innovation models such as

[11] For a similar consumer choice problem in a R&D-driven growth model, see Foellmi et al. (2014).

ours (as opposed to process-innovation models, where productivity improves over time), the quality index is the sole variable displaying a strictly positive growth rate along the balanced growth path, resulting in a positive growth of the consumer's utility.

In the rest of the economy presentation, we will now focus on a "multi-quality firms" equilibrium, i.e. an equilibrium in which incumbent firms invest a positive amount in R&D. While we take the existence of such an equilibrium for granted in the rest of this section, we will clearly discuss the parameter conditions guaranteeing its existence and uniqueness in section 4. Also, the analysis carried out in this article pertains to the balanced growth path (BGP) properties of such a model, along which all variables remain constant or grow at a constant rate. Even though this BGP will only be formally defined in section 4, from now on we will omit the functional dependance of the different variables on time, so as to simplify notations.

3.2 Market structure and pricing

Within each industry $s \in [0,1]$, firms carry out R&D in order to improve the quality of the final consumption good s. Two types of firms have the possibility to engage in R&D races: the current quality leader (incumbent), and followers (challengers). Within each sector s, each type of firms chooses an optimal amount to invest in R&D, with the probability to win the next innovation race linearly increasing along the amount invested (cf subsection 2.3 for a full description of the R&D technology). Since firms carry out those investment decisions *considering* their expected profits in the case of a successful innovation, we proceed by backward induction, first detailing in this subsection the optimal pricing of *successful* innovators, *and then* determining the corresponding optimal investment in R&D in the next one.

The market for quality goods is non-competitive. Labor is the only input, with constant unit labor requirement $a < 1$.[12]

The quality goods being characterized by unit consumption and fixed quality increments, firms use prices as strategic variables. Firms know the shares of groups P and R in the population, the respective incomes y_R and y_P as well as the preference structure of the consumers, but *cannot distinguish individuals by income*. In order to describe the strategic decisions operated by firms within a given industry, it proves convenient to define the "threshold" price $p^T_{\{j,j-m\}}(i,s)$ for which a consumer belonging to group i is indifferent between quality $j-m$ and quality j in industry s, *given* the price $p_{j-m}(s)$ charged for quality $j-m$. Determining such a threshold price amounts to solving the following equality, immediately derived from condition (7):

$$\mu_i k^j - p^T_{\{j,j-m\}}(i,s) = \mu_i k^{j-m} - p_{j-m}(s) \tag{9}$$

Considering the fact that $q_j = k^m q_{j-m}$, solving for $p^T_{\{j,j-m\}}(i,s)$ in the above equality

[12] Given the model assumes unit consumption of the quality goods, a necessarily has to be inferior to 1.

yields:[13]

$$p^T_{\{j-m,j\}}(i,s) = k^{j-m}(k^m - 1)\mu_i + p_{j-m}(s) \tag{10}$$

The price $p^T_{\{j-m,j\}}(i,s)$ is the maximum price that the firm selling the quality j in industry s can charge to a type i consumer in order to have a positive market share, when competing against the firm selling the quality $j-m$. As one can see, this threshold price positively depends on the willingness to pay for one unit of quality of type i consumers $\mu_i = \frac{1}{\lambda_i Q_i}$ (with $\mu_R > \mu_P$), as well as on the price charged by the competitor $p_{j-m}(s)$.

Having defined this threshold price, it is possible to establish the following lemma:

Lemma 1: *Within each industry $s \in [0,1]$, if $p_j(s) \geq wa$ holds for the price of some quality q_j, then for the producer of any higher quality q_{j+m}, $1 \leq m \leq n(s) - j$, there exists a price $p_{j+m}(s) > wa$ such that:*
(i) *any consumer prefers quality q_{j+m} to q_j,*
(ii) *he makes strictly positive profits.*

Proof: Considering (10), it is straightforward that $p^T_{\{j+m,j\}}(i,s) > p_j(s)$. Hence, it is always possible for the producer of the quality $j+m$ to set a price $p_{j+m}(s) > p_j(s) \geq wa$ such that $p_{j+m}(s) \leq p^T_{\{j+m,j\}}(i,s)$, i.e. such that quality q_{j+m} is preferred to quality q_j by the consumers of group i. □

Hence, within each industry s, if we take for granted that a producer never sells its quality at a price below the unit production cost wa, it is always possible for the producer of the highest quality to drive all of its competitors out of the market while still making strictly positive profits. Along this result, any firm entering the industry s with a new highest quality $q_n(s)$[14] has to consider the following trade-off concerning the pricing of its product: **setting the highest possible price for any given group of costumers, vs. lowering its price in order to capture a further group of consumers**.

It is then possible to show that in an economy characterized by two distinct groups of consumers (R and P), we have:

Lemma 2: *Within each sector $s \in [0,1]$, we have that at equilibrium,*
(1) *The highest quality is produced,*
(2) *At most the two highest qualities $q_n(s)$ and $q_{n-1}(s)$ are actually produced,*

Proof: As mentioned in the last paragraph of subsection 2.1., we focus in this article on parameter cases where growth occurs within the economy, i.e. where quality goods are consumed in at least a fraction of the continuum of sectors $[0,1]$.[15] We hence postulate

[13] We resort to the assumption (classic in the monopolistic literature) that firms within a particular sector s take the economy-wide willingness to pay for one unit of quality μ_i as given in their decision-making. Indeed, because of the existence of a *continuum* of quality good industries, firms within a given sector are "small in the big, but big in the small" (Neary, 2009): even though they resort to strategic pricing within their own industry, they do not take into account the impact of their pricing decisions on economy-wide variables such as λ_i and Q_i.

[14] Along the definition of the quality ladder in a given sector provided in subsection 2.1., $n(s)$ indeed designates the total number of innovations that took place so far within a given sector s.

[15] As also mentioned in this last paragraph, while we take for granted that we are under such parametric conditions in section 2, we clearly detail those conditions in section 3.

that there exists at least one sector s in which the quality good is consumed along the BGP, i.e. in which we necessarily have $\mu_i \geq wa$ (μ_i being the willingness to pay for a *single unit* of quality, and wa being the unit production costs, i.e. the lowest price the producer of any quality can charge). (7) also entails that the price for which the consumer prefers to buy a given quality in a sector s rather than no quality at all is increasing along the quality level. Since the production costs wa are similar for any quality level in any sector at any point in time, it means that it is either profitable to produce *every* existing quality in *every* sector, or it is never profitable to produce any quality. Along lemma 1, higher qualities drive out lower ones: *provided we are under parametric conditions that guarantee growth within the economy*, the producer of the highest quality within each sector s will hence always be able to fix a price such that (i) its quality is preferred to any other by a given consumer group i; (ii) it makes positive profits. Furthermore, since there are only two consumer groups within the economy, at most two distinct qualities can be sold within each sector. This ends the proof. □

As it can be seen from lemma 2, two different situations are possible for the equilibrium market structure and associated prices within each industry $s \in [0, 1]$: either only the top quality good $q_n(s)$ is sold to both groups of consumers (groups P and R), *or* the top quality good is sold only to the rich consumers (group R) while the second-best quality good is sold to the poor consumers (group P). Lemma 1 shows that the **decision regarding the market structure** belongs to the producer of the highest quality $q_n(s)$, considering that he is always able to set a price that will drive its competitors out. The pricing structure resulting from this optimal decision then depends on two factors: (i) the **deterministic** extent of inequality within the economy, and (ii) the result of the latest **stochastic** innovation race, where the winner (who is also the producer of the highest quality good) is either a former incumbent or a challenger.

More precisely, each industry $s \in [0, 1]$ fluctuates between two states over time, with its position being determined by the identity of the winner of the last innovation race. The two possible states (SC) and (SI) can be characterized in the following way:

- **"Successful Challenger" (SC) state:** a challenger is the winner of the last R&D race, i.e. the new quality leader is *different* from the former quality leader. In that case, the new quality leader retains exclusive monopoly rights for the highest quality $q_n(s)$ only. As we will comment below, the market structure then depends on the income distribution within the economy.

- **"Successful Incumbent" (SI) state:** the former quality leader, still carrying out R&D, is the winner of the last R&D race, and hence retains exclusive monopoly rights for *both* the highest quality $q_n(s)$ and the second-best quality $q_{n-1}(s)$. The market structure is then necessarily a monopoly.

Figure 1 illustrates the fluctuations between the two possible states over time. I will now discuss the market structure as well as the prices being charged in the two existing states.

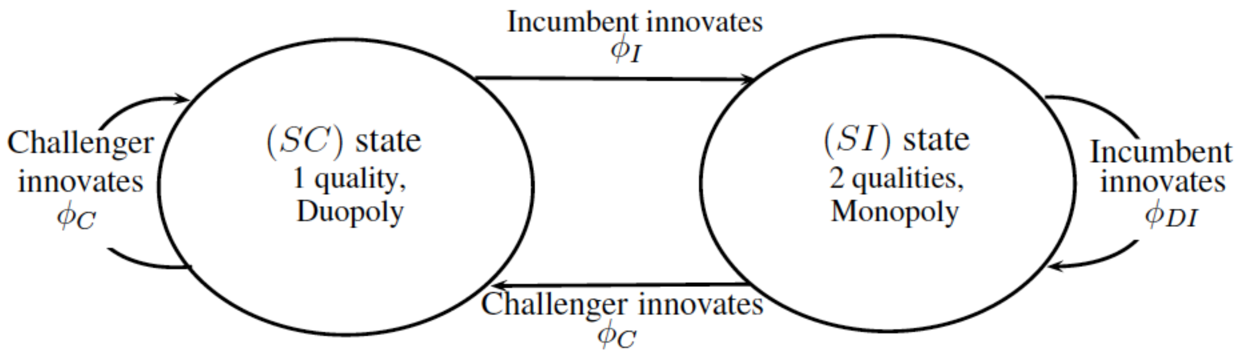

Figure 1: Two possible states

3.2.1 Prices and profits in the (SC) state

In industries being in the (SC) state, a challenger is the winner of the latest innovation race. The distance between this new leader and the "competitive fringe" (i.e. potential competitors with patent rights over lower qualities) is then of only one rung along the quality ladder. That is, even if we assume that being able to produce a quality q_j automatically grants the ability to produce any lower quality q_{j-m} ($m \in [0; j]$), the new leader will face Bertrand competition for any quality below the frontier:[16] he will hence be able to extract monopoly rents (i.e. positive profits) solely from the sale of the highest quality $q_n(s)$. One or two qualities can then be sold on the market, *depending on the pricing strategy chosen by the new quality leader* (which will itself depend on the wealth distribution in the economy). More precisely, the market structure in this state is either a **monopoly** (only quality $q_n(s)$ is sold), with the new quality leader charging a price that enables him to capture the whole market, or a **duopoly** (both qualities $q_n(s)$ and $q_{n-1}(s)$ are sold), with the new quality leader charging a higher price and serving only the upper part of the market, leaving the lower part to the producer of quality $q_{n-1}(s)$.

For the sake of exposition clarity, I will limit myself to discussing at length the resolution of the case where the equilibrium market structure in the (SC) state is a monopoly, i.e. where the income distribution makes it optimal for the new quality leader to sell the highest quality $q_n(s)$ at a price being attractive for *both* the poor and the rich consumers. Indeed, as it will become clear in the following sections, not only can the monopoly case be fully analytically solved and analyzed in terms of comparative statics, but it is also the one being robust in most parametric cases (the duopoly case is actually only a possible equilibrium under some further conditions identified in Zweimuller and Brunner, 2005). The full discussion, exposition and resolution of the duopoly case can however be found in Appendices B and E.

It is straightforward to notice that within a given industry s, charging a price guaran-

[16] Indeed, since we impose unit consumption of every quality good, firms necessarily use prices as strategic variables; also, our utility specification guarantees that different qualities are perfect substitutes (for an alternative set-up where goods are imperfect substitutes and different producers can coexist on the market selling the *same* quality, see Aghion et al., 2001).

teeing that the "poor" consumers buy the highest quality $q_n(s)$ automatically ensures that the rich consumers will consume the highest quality too, since $p^T_{\{n,n-1\}}(i,s)$ is increasing along a consumer's willingness to pay $\mu_i = \frac{1}{\lambda_i Q_i}$. It then immediately follows that the optimal price chosen by a quality leader willing to capture the whole market is $p^T_{\{n,n-1\}}(P,s)$. Assuming that the producer of quality $q_{n-1}(s)$ engages in limit pricing (i.e. $p_{n-1}(s) = wa$) and using (6) so as to obtain $\mu_i = \frac{c_i}{Q_i}$, the price $p_{SC}(s)$ being charged by the quality leader in a sector s being in the (SC) state is then of the form:

$$p_{SC}(s) = k^{n(s)} \left(\frac{k-1}{k}\right) \frac{c_P}{Q_P} + wa \tag{11}$$

The profits $\pi_{SC}\big(n(s)\big)$ of a successful challenger in an industry s where there have been $n(s)$ successful innovations so far are then of the form:

$$\pi_{SC}\big(n(s)\big) = k^{n(s)} L \left(\frac{k-1}{k}\right) \frac{c_P}{Q_P} \tag{12}$$

3.2.2 Prices and profits in the (SI) state

In an industry being in the (SI) state, the former quality leader has won a second R&D race in a row, and retains exclusive monopoly rights for *both* the highest quality $q_n(s)$ and the second-best quality $q_{n-1}(s)$. According to lemma 2, the market structure is then necessarily a *monopoly*; however, unlike the monopoly case in the (SC) state, the two highest qualities both have positive market shares. Indeed, the quality leader is two rungs above the competitive fringe along the quality ladder: facing two groups of consumers having different levels of income, he will hence be able to offer two distinct price-quality bundles so as to maximize its profit (Mussa and Rosen, 1978). The price charged by the monopolist for its second-best quality $q_{n-1}(s)$ will be the maximal price enabling him to capture the poor group of consumers $p^T_{\{n-1,n-2\}}(P,s)$, given that the producer of quality $q_{n-2}(s)$ engages in limit pricing. Denoting this price by $p^P_{SI}(s)$, we have:

$$p^P_{SI}(s) = k^{n(s)-1} \left(\frac{k-1}{k}\right) \frac{c_P}{Q_P} + wa \tag{13}$$

The price charged for the highest quality $q_n(s)$ will then be $p^T_{\{n,n-1\}}(R,s)$, given the price $p^P_{SI}(s)$ charged for quality $q_{n-1}(s)$. Denoting this price by $p^R_{SI}(s)$, we have:

$$p^R_{SI}(s) = k^{n(s)} \left(\frac{k-1}{k}\right) \frac{c_R}{Q_R} + k^{n(s)-1} \left(\frac{k-1}{k}\right) \frac{c_P}{Q_P} + wa \tag{14}$$

The profits $\pi_{SI}\big(n(s)\big)$ of a successful incumbent in an industry s where there have been $n(s)$ successful innovations so far are then of the form:

$$\pi_{SI}\big(n(s)\big) = k^{n(s)} L \left[(1-\beta)\left(\frac{k-1}{k}\right) \frac{c_R}{Q_R} + \left(\frac{k-1}{k^2}\right) \frac{c_P}{Q_P}\right] \tag{15}$$

3.2.3 Quality consumption indices

Denoting by θ_{SC} and θ_{SI} the shares of sectors being respectively in the (SC) and the (SI) state, it is finally possible to notice that the "quality consumption indices" Q_P and Q_R take the following form in the case we have a monopoly in the (SC) state:

$$Q_P = \int_{\theta_{SC}} k^{n(s)} ds + \int_{\theta_{SI}} k^{n(s)-1} ds; \qquad Q_R = \int_0^1 k^{n(s)} ds \tag{16}$$

3.3 The R&D sector

Within each industry $s \in [0,1]$, firms carry out R&D in order to discover the next quality level. Two types of firms have the possibility to engage in R&D races: the current quality leader (incumbent), and followers (challengers). We assume free entry, with every firm having access to the same R&D technology within each sector s. Innovations are random, and occur for a given firm f within sector s according to a Poisson process of hazard rate $\phi_f(s)$. Labor is the only input, and we assume constant returns to R&D at the firm level: in order to have an immediate probability of innovating of $\phi_f(s)$ in a sector s having reached the quality level $k^{n(s)}$, a firm needs to hire $F\frac{k^{n(s)+1}}{Q}\phi_f(s)$ units of labor, F being a positive constant and $Q = \int_0^1 k^{n(s)} ds$ being the economy's "quality index" (i.e. the average quality level being reached across sectors). This R&D cost function implies that R&D becomes more difficult in sectors that are too much ahead of the "average technology" being reached in the rest of the economy.

As it becomes clear when considering the sector-specific profit functions (12) and (15), such a cost structure ensures that innovations become neither more profitable in sectors where there have already been more quality jumps, nor less profitable as the quality index grows: while the first case would ultimately lead to the disappearance of every sector but the most performant (and hence profitable) one, the second would lead to a no-growth steady-state.[17] More precisely, our R&D sector specification guarantees that within the group of industries being in a given state, the probability to innovate for a given type of firm (challengers or incumbents) will be the same in every sector, **regardless of the sector-specific rung $n(s)$ having been reached along the quality ladder**. This will lead to the survival of every quality sector along the BGP, and guarantees that every industry is symmetric with respect to transition probabilities from one state to the other. For the sake of notation brevity, we hence drop the sector dependance in the rest of this subsection.

For a given sector s in which n innovations have occurred so far, we define $v_C(n)$ as the value of a challenger firm, $v_{SC}(n)$ as the expected present value of a quality leader having innovated once, and $v_{SI}(n)$ as the expected present value of a quality leader having innovated twice. Free entry and constant returns to scale imply that R&D challengers

[17] This type of assumption is standard in quality-ladder models to ensure long-term balanced growth; it can be indifferently embodied in growing R&D costs for a given innovation probability *or* in a decreasing probability to innovate for given R&D costs (see Barro and i Martin (2003), Chapter 7).

have no market value, whatever state the economy finds itself in: $v_C(n) = 0$. Free entry of challengers in the successive R&D races also yields the traditional equality constraint between expected profits of innovating for the first time $\phi_C v_{SC}(n)$ and engaged costs $\phi_C \frac{k^{n+1}}{Q} wF$:

$$v_{SC}(n) = \frac{k^{n+1}}{Q} wF \tag{17}$$

The incumbent on the other hand participates to the race while having already innovated at least once, and hence being the current producer of the leading quality for industries in the (SC) state/of the **two** highest qualities for industries in the (SI) state. In industries being in the (SC) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rv_{SC}(n) &= \max_{\phi_I \geq 0} \Big\{ \pi_{SC}(n) - wF\frac{k^{n+1}}{Q}\phi_I \\ &+ \phi_I \Big(v_{SI}(n+1) - v_{SC}(n) \Big) + \phi_C \Big(v_C - v_{SC}(n) \Big) \Big\} \end{aligned} \tag{18}$$

In a (SC)-state sector, the incumbent earns the profits $\pi_{SC}(n)$ and incurs the R&D costs $wF\frac{k^{n+1}}{Q}\phi_I$. With instantaneous probability ϕ_I, the leader innovates once more, the industry jumps to the state (SI), and the value of the leader (now detaining monopoly rights over two distinct qualities) climbs to $v_{SI}(n+1)$.[18] However, with overall instantaneous probability ϕ_C, some R&D challenger innovates, and the quality leader falls back to being a follower: its value drops to $v_C = 0$. The industry then remains in the state (SC), and only one quality is produced.

In industries being in the (SI) state, the "quality-differentiated" incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rv_{SI}(n) &= \max_{\phi_{DI} \geq 0} \Big\{ \pi_{SI}(n) - wF\frac{k^{n+1}}{Q}\phi_{DI} \\ &+ \phi_{DI} \Big(v_{SI}(n+1) - v_{SI}(n) \Big) + \phi_C \Big(v_C - v_{SI}(n) \Big) \Big\} \end{aligned} \tag{19}$$

The incumbent in the (SI) state earns the profits $\pi_{SI}(n)$ of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. He incurs the R&D costs $wF\frac{k^{n+1}}{Q}\phi_{DI}$. With instantaneous probability ϕ_{DI}, the incumbent innovates once more, in which case its value becomes $v_{SI}(n+1)$.[19] Hence, the incumbent will still be the producer of the two qualities being sold, *but* he will drive himself out of the market for the former quality q_{n-1}, that has become quality q_{n-2} with the latest quality jump. The industry then remains in state (SI). With instantaneous probability ϕ_C,[20] some R&D follower innovates, and the quality leader then falls back to being an

[18] Accordingly to the crucial condition identified and discussed in the introduction as being necessary so as to to generate innovation by incumbent, this expected value of innovating for a second time $v_{SI}(n+1)$ is different from the expected value of innovating for the first time $v_{SC}(n)$.

[19] We have indeed established with Lemma 2 that at most two successive quantities are sold at equilibrium.

[20] The challengers will invest the same amount in the R&D sector ϕ_C whether the considered sector is in state (SC) or (SI), since they face the same expected reward $v_{SC}(n+1)$ in both cases: a successful

R&D challenger: its value falls to $v_C = 0$. The industry then jumps to the state (SC), and only the new highest quality is sold by the latest successful innovator.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. (18) and (19) then yield the following first-order conditions:

$$\left(-wF\frac{k^{n+1}}{Q} + v_{SI}(n+1) - v_{SC}(n)\right)\phi_I = 0, \quad \phi_I \geq 0 \tag{20}$$

$$\left(-wF\frac{k^{n+1}}{Q} + v_{SI}(n+1) - v_{SI}(n)\right)\phi_{DI} = 0, \quad \phi_{DI} \geq 0 \tag{21}$$

For industries being in the (SC) state, (20) yields a relationship between the R&D costs and the incremental value of a further innovation. Combined with (20), (21) entails either $\phi_{DI} = 0$ or $v_{SC}(n) = v_{SI}(n)$. The second possibility cannot be true, since $\pi_{SI}(n) > \pi_{SC}(n)$: we hence necessarily have that $\phi_{DI} = 0$. Plugging (17), (20) and (21) in (18) and (19) and substituting for the profit values obtained in (12) and (15), it is possible to obtain the 2 following expressions, equating incurred R&D costs and expected profits in both possible states:[21]

$$\frac{wF}{Q} = \frac{L\left(\frac{k-1}{k}\right)\frac{c_P}{Q_P}}{r+\phi_C} \tag{22}$$

$$\left(\frac{k+1}{k}\right)\frac{wF}{Q} = \frac{L\left[(1-\beta)\left(\frac{k-1}{k}\right)\frac{c_R}{Q_R} + \left(\frac{k-1}{k^2}\right)\frac{c_P}{Q_P}\right]}{r+\phi_C} \tag{23}$$

4 Balanced growth path equilibrium

4.1 Labor market equilibrium

We first move to characterizing the equilibrium on the labor market. While challengers invest a total amount of $F\frac{k^{n(s)+1}}{Q}\phi_C$ in R&D in every industry s, incumbents only invest the amount $F\frac{k^{n(s)+1}}{Q}\phi_I$ in industries being in the (SC) state. The total labor demand in the R&D sector is hence equal to $F\left(\int_0^1 \frac{k^{n(s)+1}}{Q}\phi_C ds + \int_{\theta_{SC}} \frac{k^{n(s)+1}}{Q}\phi_I ds\right)$. Unit consumption of the differentiated goods and identical marginal costs of production regardless of the quality level yield a total amount of aL units of labor being devoted to the production of the quality goods. Finally, $(L/w)(\beta c_P + (1-\beta)c_R)$ are the units of labor being devoted to the production of the standardized good.

The following equation then describes the equilibrium on the labor market:

$$L = Fk\phi_C + Fk\phi_I\frac{\int_{\theta_{SC}} k^{n(s)}ds}{Q} + aL + (L/w)(\beta c_P + (1-\beta)c_R) \tag{24}$$

innovation by a challenger indeed always brings the industry back to state (SC).

[21] Considering (22) and (23), it is also confirmed that the conditions ruling the R&D investment decisions within one sector don't depend on any sector-specific value, but only on economy-wide variables: as announced in the beginning of the subsection, the probabilities to innovate are hence the same in every sector.

4.2 Balanced growth path analysis

Definition 1 *In the case we have a monopoly market structure in the (SC) state, an equilibrium is defined by a time path for consumption of the homogenous good for both types of consumers* $\{c_i(t)\}_{i=(R,P),t=0}^{\infty}$ *that satisfies* (6), *a time path for the "quality index" for both types of consumers* $\{Q_i(t)\}_{i=(R,P),t=0}^{\infty}$ *that satisfies* (7), *time paths for innovation probabilities (and corresponding sector-specific R&D expenditures) by incumbents and challengers* $\{\phi_C(t), \phi_I(t)\}_{t=0}^{\infty}$ *that satisfy* (17) *and* (20), *time paths for sector-specific prices and profits* $\{p_{SC}(s,t), p_{SI}^{P}(s,t), p_{SI}^{R}(s,t), \pi_{SC}\big(n(s,t)\big), \pi_{SC}\big(n(s,t)\big)\}_{s\in(0,1),t=0}^{\infty}$ *that satisfy* (11), (13), (14), (12) *and* (15), *and a time path of the interest rate* $\{r(t)\}_{t=0}^{\infty}$ *which satisfies* (8).

In addition, we define a balanced growth path (BGP) as an equilibrium path along which every variable grows at a constant rate, either null or positive. In such a *product*-innovation model (i.e. precluding any productivity improvement) with fixed wage and population levels w and L, the BGP is characterized by constant levels of innovation ϕ_C and ϕ_I, overall wealth Ω and consumption c_i ($i = R, P$).[22] Consumers however still become better-off over time due to the quality improvements of the differentiated goods and the resulting growth of individual utility. As already stated in the previous section, we focus in this paper on such a BGP, and we now proceed to describing its properties.

4.2.1 Residual consumption levels c_P and c_R along the BGP

The R&D sector as well as the labor market equilibrium conditions (22), (23) and (24) depend on the "residual" consumption of the standardized homogenous good c_i,[23] which we will now further characterize. First, constant levels of consumption imply that along the BGP, current income y_i equates current consumption $c_i + P_i$, i.e. we have $c_i = y_i - P_i$ once we have reached the long-run growth path.

Using (11), (13) and (14) as well as the BGP properties of the random process governing the fluctuations between the two possible states (SC) and (SI) within every industry, it is then possible to obtain the following expressions for the economy-wide price indices P_R and P_P (see Appendix A for the demonstration and detailed computations):

$$P_P = \frac{(k-1)y_P + kwa}{2k-1} \tag{25}$$

$$P_R = \frac{(2k-1)(k-1)\theta_{SI} y_R + (k-1)k y_P + k^2 wa}{(2k-1)(k+\theta_{SI}(k-1))} \tag{26}$$

As we can see considering (26), the economy-wide price index P_R depends on the share θ_{SI} describing the proportion of industries being in the (SI) state. This stems from

[22]The consumption of the continuum of quality-differentiated goods is anyway always constant, since we impose unit consumption in this model.

[23]In the case of the R&D sector equilibrium conditions, it is through the willingness to pay $\mu_i = \frac{c_i}{Q_i}$.

the fact that rich consumers pay *less* than their maximum threshold price in (SC)-state industries, while being efficiently price-discriminated (and hence paying a higher price) in (SI)-state industries. More precisely, we have $\frac{\partial P_R}{\partial \theta_{SI}} = \frac{k(k-1)\big((2k-1)y_R-(k-1)y_P-kwa\big)}{(2k-1)(k+(k-1)\theta_{SI})^2} > 0$: quite intuitively, P_R is higher for a higher proportion of industries being in the (SI) state; as a consequence, the willingness to pay for one unit of quality $\mu_R = \frac{y_R-P_R}{Q}$ is on the other hand decreasing along θ_{SI}. For some intuitions regarding the impact of inequality on the long-run growth rate (cf. next section), it is finally also interesting to notice that within a *specific* sector s being in the (SI) state, the price p_{SI}^R that the incumbent will be able to charge to the rich is *decreasing* along θ_{SI} (since p_{SI}^R as presented in (14) depends positively on μ_R): the fewer industries in the (SI) state, the higher the price the leaders will be able to charge to the rich consumers *within those industries*.

P_P on the other hand does not depend on the shares θ_{SC} and θ_{SI}: indeed, the poor consumers pay their maximum threshold price in every industry.

Using (25) and (26), it is then finally possible to obtain the following expressions for the homogenous good consumption of both consumer groups c_P and c_R:

$$c_P = \left(\frac{k}{2k-1}\right)(y_P - wa) \tag{27}$$

$$c_R = \frac{k\big((2k-1)y_R-(k-1)y_P-kwa\big)}{(2k-1)(k+(k-1)\theta_{SI})} \tag{28}$$

4.2.2 Industry shares along the BGP

Along the BGP equilibrium, it is possible to express θ_{SC} and θ_{SI} as functions of the innovation rates ϕ_C and ϕ_I. Indeed, so as to ensure that c_R as expressed in (28) remains constant, the share of industries being in each state must remain constant. Hence, the flows in must equal the flows out of each state: we then have the condition $\phi_C\theta_{SI} = \phi_I\theta_{SC}$ that has to be respected along the BGP.[24] Combining it with the fact that the two shares sum up to 1 (i.e. $\theta_{SC} + \theta_{SI} = 1$), we obtain:

$$\theta_{SC} = \frac{\phi_C}{\phi_I+\phi_C}, \quad \theta_{SI} = \frac{\phi_I}{\phi_C+\phi_I} \tag{29}$$

4.2.3 Labor market equilibrium along the BGP

The BGP properties of the random process governing the fluctuations between the two possible states (SC) and (SI) within every industry (cf. Appendix A for a full exposure) make it possible to obtain the following expression for the long-run labor market equilibrium condition:

$$L = Fk\phi_C + \theta_{SC}Fk\phi_I + aL + (L/w)\left(\beta c_P + (1-\beta)c_R\right) \tag{30}$$

[24] Indeed, for each industry being in the (SC) state, the probability to exit this state is equal to the probability $\phi_I(s)$ of an incumbent innovating; for each industry being in the (SI) state, the probability to enter the (SC) state corresponds to the probability $\phi_C(s)$ of a challenger innovating.

4.2.4 Existence and uniqueness of the BGP

Having now fully described the long-run properties of our economy, we can move to characterizing the parametric conditions ensuring the existence and uniqueness of a "multi-quality firms BGP".

Proposition 1 (Existence and uniqueness of a steady state equilibrium): *For k and β sufficiently high and for not too low values of d, there exists a unique BGP along which (i) we necessarily have a monopoly in the (SC) state, (ii) both incumbents and challengers invest strictly positive amounts in R&D, and (iii) the economy-wide quality index grows at the constant rate* $\gamma = \frac{\dot{Q}}{Q} = (k-1)\phi_C(1+\frac{\phi_I}{\phi_I+\phi_C})$.[25]

Proof: cf Appendix C. □

Note that Proposition 1 implies not only that there exists a unique positive solution for the system of variables as defined in Definition 1, but also that the equilibrium with a monopoly market regime is robust (existence) while its duopoly counterpart is not (uniqueness). While the full demonstration of this proposition is available in Appendix C, I will here provide intuitions regarding the conditions on the exogenous parameters needed so as to obtain this result.

A first condition guarantees a strictly positive amount invested in R&D by incumbents along the BGP, and this is that k needs to be sufficiently high. Indeed, k represents the utility increment of consuming quality q_n over quality q_{n-1}: the higher k, the higher the gap between p_{SC} and p_{SI}^R in any given industry. In other words, high values of k ensure that the gains from price-discriminating are high enough to represent viable incentives for the incumbent to invest in R&D.

The second two conditions (β sufficiently high and d not too low) are needed so as to guarantee the existence and uniqueness of the obtained BGP. Regarding the existence, we indeed need to check that for the obtained equilibrium values of the endogenous variables, the monopoly market structure in the (SC) state is *robust*, i.e. the new leader does not prefer the alternative regime when comparing expected profits. Regarding the uniqueness, we also have to make sure that the equilibrium values obtained when solving for a BGP with a duopoly market structure in the (SC) state (as defined in Appendix B) do not define a robust equilibrium. Intuitively enough, high values of β and not too low values of d ensure that the monopoly is the **only** viable price regime: indeed, a leader facing an important group of poor people (both in terms of size and in terms of purchasing power) is not going to be willing to abandon that part of the market to its direct competitor.

A "multi-quality firms BGP" with a monopoly market regime in the (SC) state hence emerges **only** for parameter values respecting the conditions stated in Proposition 1 above. Appendices B and E similarly define conditions under which there exists a robust "multi-

[25] The BGP times paths of the quality consumption indices Q_P and Q_R can be directly derived from the growth rate of the quality index. We have $\frac{\dot{Q}_R}{Q_R} = \frac{\dot{Q}}{Q}$; on the other hand, along (16) we have $\frac{\dot{Q}_P}{Q_P} = (k-1)\phi_C\left(1+\frac{\phi_I}{\phi_C+(1/k)\phi_I}\right)$.

quality firms BGP" with a duopoly market regime in the (SC) state.[26] Outside those parameter constellations though, condition (20) yields $\phi_I = 0$, and the model collapses to a multi-industry version of the Zweimuller and Brunner (2005) framework.

Proposition 1 states that in an economy where sufficiently strong disparities in purchasing power exist, incumbents have an incentive to keep investing in R&D beyond their first successful innovation. In our framework, the immediate consequence of this result is the **endogenous emergence of multi-quality leaders in a dynamic quality-ladder model**, since income disparities generate both (1) the survival of more than one quality at the equilibrium, and (2) positive investment in R&D activities by incumbents.

A few further comments can be made. First, a salient implication of this result is the existence of *demand-related* determinants of innovation by incumbents. Here, positive investment in R&D by quality leaders is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers, as well as without any concavity in the R&D cost function. This model therefore hints at the existence of so far overlooked incentives for innovation by incumbent stemming from the demand structure rather than from the supply side (i.e. R&D sector characteristics and R&D capabilities of challenger and incumbent firms), and opens the field for further investigations.[27] Second, this result emphasizes the macroeconomic consequences of the negative "heterogenous taste for quality" externality identified by Mussa and Rosen (1978) in a micro framework. This externality can be formulated in the following way: in the absence of the possibility of first-degree discrimination, the existence of "poor" consumers prevents the monopolist from capturing the maximum costumer surplus from those who have a stronger taste for quality. In a static framework, a multi-quality monopolist internalizes this negative externality by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units. As a consequence, a wider range of qualities than what would be optimal is finally offered. In our dynamic model with endogenous innovation, the monopolist only retains exclusive patent rights for as many qualities as R&D races he has won: the negative externalities stemming from having to serve two distinct groups of consumers having different quality valuations is then internalized by expanding the line of product towards *higher* (and not lower) qualities, i.e. through R&D investment.

5 Distribution of income and long-term growth

We now investigate the implications of such a model regarding the existing interactions between income distribution and long-run growth operating through the demand market.

[26]However, due to the impossibility to obtain closed-form solutions, we have to resort to simulations so as to determine the robustness conditions of the BGP in the case of a duopoly in the (SC) state.

[27]As we already pointed out in the introduction, Aghion et al. (2001) had already provided a quality-ladder framework in which it was possible to investigate the influence of product market competition on innovation intensity. However, they did not allow for free entry of firms, only allowing for the existence of two active firms in the R&D sector.

In their contribution, Zweimuller and Brunner (2005) had argued that in a vertical differentiation framework featuring non-homothetic preferences and heterogenous consumers, a rising level of inequality *systematically* decreases the R&D investment rate of challengers. This result led them to conclude to an unambiguous detrimental impact of inequality (whether it stems from higher income gaps or greater wealth concentration) on long-term growth. However, as already stated before, their model pins down the R&D investment rate with a simple free entry condition in the R&D sector, overlooking the possibility of incumbents investing in R&D. As we will now show below, taking into account the possibility of second-degree price discrimination and the resulting participation of incumbents to innovation races totally modifies the predictions of the model regarding the impact of varying inequality on R&D investment and the resulting long-run growth rate.

In the following analysis, we consider two types of variations in the extent of wealth disparities: (a) a larger income *gap* (i.e. a decrease in d for a fixed level of β), and (b) a greater wealth *concentration* (i.e. an increase in β for a given d). The results of those comparative statics can be summarized in the following proposition:

Proposition 2 (Wealth distribution and long-term growth):
Under the parametric conditions guaranteeing the existence of a unique BGP with a monopoly market structure in the (SC) *state, we have the following comparative statics for varying values of* β *and d:*

- *(a) Effect of a larger income gap (corresponding to a decrease in d): the challengers' innovation rate* ϕ_C *as well as the incumbent's innovation rate* ϕ_I *increase, resulting in an increase of the long-run growth rate* γ*. An increase in the income gap also leads to a greater share of R&D activities to be carried out by incumbents.*
- *(b) Effect of a greater income concentration (corresponding to an increase in* β*): the challengers' innovation rate* ϕ_C *as well as the incumbent's innovation rate* ϕ_I *decrease, leading to a decrease of the long-run growth rate* γ*. An increase in income concentration also leads to a greater share of R&D activities to be carried out by challengers.*

Proof: cf Appendix D. □

(a) Let us first comment the effects of a larger income gap, i.e. of a decrease in d.

So as to obtain intuitions regarding the variations in the R&D investment rates following a variation in the income gap, we consider the impact of such a shock on the expected gains associated to successfully innovating for the first and the second time. One can first notice that since we keep both β and the quantities produced fixed (the quality-differentiated industries face unit consumption), there can be no variation in the *market size* following an increase in the income gap: profit variations will derive from *price* adjustments.

We first comment the variation of ϕ_I, i.e. of the amount invested in R&D by the current incumbent operating in a (SC)-state industry. For this non-differentiated leader, the critical income when choosing how much to invest in R&D is the one of rich households, since the *incremental* gain from innovating for a second time stems from the higher price charged to the upper end of the market. The income of rich households increases following the considered shock: hence, at a given level of wealth Ω, a decrease in d (i.e. a redistribution of wealth from the poor to the rich) has a *positive price effect* on the profits of a successful incumbent. The incentives to invest in R&D for an incumbent have hence become greater: ϕ_I increases.

The variation in ϕ_C is somewhat more difficult to rationalize a priori, since the exact counterpart of the above reasoning points to a *negative price effect* on the profits of a successful challenger: indeed, following a decrease in d, a successful challenger entering a given industry with only one quality at its disposal has to charge a *lower* price so as to capture the impoverished lower end of the market. This negative price effect is however (at least partly) counteracted by a less obvious positive price effect, linked to the variation in the share of industries θ_{SI} being in the (SI) state. θ_{SI} indeed increases following the increase in ϕ_I commented above. In sectors being in the (SI) state, poor consumers are being sold quality $n-1$, that is one rung away from the industry frontier. As a consequence, they tend to value more the fewer industries in which they are being sold the highest available quality, i.e. the decreased share of sectors being in the (SC) state. Successful challengers benefit from this greater appeal. This positive price effect is being captured by the ratio Q/Q_P, present in the free-entry R&D condition (22) and increasing along θ_{SI}.[28] One should also note that part of the expected profits considered by the challengers when entering their first innovation race pertains to the possibility to price-discriminate *if* they innovate for a second time. The increase of those potential second-round profits (cf above the comments regarding the increase in ϕ_I), along with the identified positive price effect, finally lead the R&D free-entry conditions to pin down the challengers' innovation rate ϕ_C at a higher level than before the shock on d.

(b) We now move to commenting the effects of an increase in β when we have a monopoly price regime in the (SC) state. I first note that a rise in the share of the population being poor β while keeping d constant corresponds to a higher concentration of wealth among a smaller group of rich people. Indeed, it implies an increase in the *relative* income of a rich consumer ($\frac{\partial d_R}{\partial \beta} = \frac{1-d}{(1-\beta)^2} > 0$): there are more poor with the same income, and fewer rich with more income. As we can see in Proposition 2, this type of variation in the inequality level is unequivocally detrimental for economic growth. We now comment the intuitions pertaining to the variations of the different variables.

Once again, we first comment the impact of an increase of wealth concentration on the R&D investment of incumbents ϕ_I. Following an increase in β (and unlike a shock on d which generates only variations in prices), we have both a market size *and* a price effect on the expected profits of a successful incumbent. Indeed, price-discriminating monopolists operating in (SI)-state industries can now charge a *higher* price, but to a *smaller* part of

[28]More precisely, we have $Q/Q_P = \frac{k}{k\theta_{SC}+\theta_{SI}}$, which can be transformed into $Q/Q_P = \frac{k}{k-(k-1)\theta_{SI}}$ using the property $\theta_{SC} + \theta_{SI} = 1$.

the population. Contrarily to what happens in the horizontal differentiation case (Foellmi and Zweimuller, 2006), the negative market size effect systematically dominates here, and the incumbents' investment in R&D ϕ_I decreases. This difference between horizontal- and vertical-differentiation models can be rationalized through the fact that in the case of vertical differentiation, the price effect is limited by the presence of a competitive fringe, which is not present in the case of horizontal differentiation.

Regarding variations of ϕ_C following a shock on β, we first notice that the market size of a successful challenger is not altered by such a shock. The decrease in ϕ_C following an increase of the wealth concentration then stems from (i) the negative price effect resulting from a decrease in the ratio Q/Q_P following the decrease in ϕ_I, and (ii) the decrease of the potential profits realized if the successful challenger innovates for a second time.

Several conclusions can be derived from the results presented in this section.

First, when asking "How does inequality affect investment in R&D and growth in a quality-ladder set-up?", the answer depends crucially on whether higher levels of inequality result from a larger income gap or from a higher income concentration. In the case of a larger income gap, only price effects are at play, leading to an increase in the R&D investment of both types of actors, and ultimately to a higher economy's growth rate. In the case of an increased wealth concentration on the other hand, the positive price effect is more than counterbalanced by a negative market size effect, leading to a decrease in the R&D investments of both types of actors. The two different shocks also lead to different predictions in terms of reallocation of the overall R&D bulk from one type of actor to another: while a greater income gap leads to a greater share of the overall R&D being carried out by incumbents, the reverse is true in the case of a higher wealth concentration.

Second, when comparing those predictions to the ones obtained in the case of expanding-variety growth models (Foellmi and Zweimuller, 2006; Foellmi et al., 2014), we see that the nature of the differentiation considered (i.e. horizontal vs vertical) is crucial in order to predict the impact of varying inequality on R&D investment and growth. Indeed, Foellmi and Zweimuller (2006) have shown that in an horizontal differentiation framework, higher levels of inequality are systematically positive for an economy's rate of growth. The intuition pertains to a product's life-cycle: lower levels of inequality induce a positive market size effect (the market for a new good develops faster into a mass market), but a negative price effect (the willingness to pay for a new product decreases with a less wealthy rich class). The latter always dominates the former, since profit flows *early* in the product's life cycle matter more, and are lowered by a decrease in inequality. Foellmi et al. (2014) show that even when the monopolist can engage in process innovation so as to transform its luxury good into a product of mass consumption (hence engaging in a form of price discrimination), higher inequality levels still have a positive impact on growth *provided* the technological spillovers stemming from the introduction of mass production are not too important.

The mechanisms present in those two models however rely on the crucial assumption that a firm keeps *permanent monopoly rights* over a given good, without running the risk of being leapfrogged. In the case of a vertical-differentiation model where the introduction of new products pushes the older ones further from the frontier, the predictions are altered. As it was possible to demonstrate in this section, higher levels of wealth concentration are detrimental for growth in a quality-ladder framework: the positive price effect is dominated by the negative market size effect. Also in the case of a higher income gap, the mechanisms leading to a higher growth rate are fundamentally different in the two frameworks: in a quality-ladder model, the positive impact on economic growth is obtained *despite* a negative price effect on a new entrant's profits, and mainly through a reallocation of R&D activities from challengers to incumbents.

Finally, those results show how decisive it is to take into account the behavior of incumbents when analyzing the interactions of aggregate demand and long-run growth in a quality-ladder model. Indeed, including incumbents in the analysis leads us to totally overturn the conclusions obtained in Zweimuller and Brunner (2005). More precisely, in the case we have a monopoly in the (SC) state, the model presented here yields opposite predictions regarding the impact on the overall growth rate of a decrease in d, and predicts a negative impact of an increase in β while their model finds none.[29] This framework also makes it possible to further characterize the evolution of the *allocation* of overall R&D expenditures between challengers and incumbents.

6 Conclusion

This paper contributes to the analysis of the interactions between income distribution and long-term growth operating through the demand side. It first demonstrates that disparities in purchasing power justify investment in R&D by both leaders and challengers, providing a demand-driven rationale for innovation by incumbents. Indeed, the strictly positive innovation rate of the incumbent is here obtained with constant returns to R&D efforts and without any advantage of the incumbent in the R&D field (supply side), by allowing for income inequality to generate different quality valuation of poor and rich consumers (demand side). The paper then also provides a significant contribution to the literature investigating the impact of income inequality on growth, showing that while an increase in the income gap can be beneficial for growth, a greater wealth concentration is systematically detrimental for the economy.

Some lines of further work can be quickly sketched. An obvious extension to this model would be to treat the more general case of more than two types of consumers, in order for the incumbent to keep investing in R&D after the second successful race. A model such as this one can also be applied to a two-country framework, in order to contribute to the developing literature studying the determinants and impact of vertical, intra-industrial trade (Fajgelbaum et al., 2011). Indeed, while the impact on growth of inter-industrial quality

[29] Indeed, in the case of a monopoly price regime, the size of the rich population does not matter at all, since successful challengers can never correctly price-discriminate them.

trade has already been extensively studied (product life-cycle), we believe the framework presented in this paper would be a good starting point for the elaboration of a dynamic model of intra-industrial quality trade (quality life-cycle). Finally, one could also take the model to the data so as to test its predictions. A possible identification strategy would be to test whether income distribution variations within a target market have a significant impact on *product* vertical innovation of firms, while none on *process* innovation.

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A - Computation of the price indices P_R and P_P

Poor consumers pay $p_{SC}(s)$ for goods being produced in (SC)-state industries, and $p^P_{SI}(s)$ for goods being produced in (SI)-state industries. We hence have $P_P = \int_{\theta_{SC}} p_{SC}(s)ds + \int_{\theta_{SI}} p^P_{SI}(s)ds$. Rich consumers also pay $p_{SC}(s)$ in industries being in the (SC) state, and $p^R_{SI}(s)$ in industries being in the (SI) state: we have $P_R = \int_{\theta_{SC}} p_{SC}(s)ds + \int_{\theta_{SI}} p^R_{SI}(s)ds$. Using (11), (13) and (14), we then obtain the following expressions for the two economy-wide indices:

$$\begin{aligned} P_P &= \int_{\theta_{SC}} k^{n(s)} \left(\frac{k-1}{k}\right) \frac{(y_P - P_P)}{Q_P} ds + \int_{\theta_{SI}} k^{n(s)-1} \left(\frac{k-1}{k}\right) \frac{(y_P - P_P)}{Q_P} ds \\ P_R &= \int_{\theta_{SC}} k^{n(s)} \left(\frac{k-1}{k}\right) \frac{(y_P - P_P)}{Q_P} ds + \int_{\theta_{SI}} \left(k^{n(s)} \left(\frac{k-1}{k}\right) \frac{y_R - P_R}{Q_R} + k^{n(s)-1} \left(\frac{k-1}{k}\right) \frac{y_P - P_P}{Q_P}\right) ds \end{aligned}$$

Considering the fact that along (16) we have $Q_P = \int_{\theta_{SC}} k^{n(s)}ds + \int_{\theta_{SI}} k^{n(s)-1}ds$, it is straightforward to simplify the first expression into $P_P = \left(\frac{k-1}{k}\right)(y_P - P_P)$, which itself yields:

$$P_P = \frac{(k-1)y_P + kwa}{2k-1}$$

Since $Q_R = \int_0^1 k^{n(s)}ds$, the above expression for P_R simplifies to:

$$P_R = \left(\frac{k-1}{k}\right)(y_P - P_P) + \left(\frac{k-1}{k}\right)(y_R - P_R) \underbrace{\frac{\int_{\theta_{SI}} k^{n(s)}ds}{\int_0^1 k^{n(s)}ds}}_{=(*)} + wa$$

Proposition 3: *Along the BGP (i.e. for t big enough), the number of quality jumps per unit of time $\frac{n(s,t)}{t}$ can be approximated by the same constant ν in every industry s.*

Proof: We focus on one sector s which has just jumped back from state (SI) to state (SC). The operating firm on the market is hence a former challenger. There will then be a certain number $M-1$ (with $M \in [1, +\infty[$) of "investment races" for which the considered sector remains in the state (SC), i.e. for which the winner of the race is a challenger. At some point though, the winner of the Mth race ends up to be an incumbent: the sector switches to state (SI). Since incumbents stop investing in R&D once having fully price-discriminated, the next jump will *necessarily* bring the sector back to state (SC), and a new cycle begins. We denote the number of innovations having occurred in this cycle as $W_c = M + 1$, where c is the index of the cycle. W_c can be viewed as a "reward", with S_c being the holding time until this reward is reached. The couples (S_1, W_1), (S_2, W_2),... are i.i.d. random variables. The total number of innovations at a given time t for which X_t cycles have already occurred is $Y_t = \sum_{c=1}^{X_t} W_c$. Y_t is a "renewal-reward process": it depends on the holding times S_1, S_2,... between two cycles, and on the rewards corresponding to each cycle W_1, W_2,... Along the BGP, we can then apply the strong law of large numbers for renewal-reward processes, which states that $\lim_{t\to\infty} \frac{Y_t}{t} = \frac{E[W_1]}{E[S_1]}$. Since every industry $s \in [0,1]$ displays exactly the same immediate innovation probabilities for incumbents (ϕ_I) and challengers (ϕ_C), the number of innovations $n(s,t)$ in every sector s can be described by a renewal-reward process identical to the one described above. We then have that the number of quality jumps per unit of time $\frac{n(s,t)}{t}$ is equal to the same constant ν in every industry s for t big enough, with $\nu = \frac{E[W_1]}{E[S_1]}$. This ends the proof. □

Using Proposition 3, we have $(*) = \frac{k^{\nu T} \int_{\theta_{SI}} ds}{k^{\nu T} \int_0^1 ds} = \theta_{SI}$, which leads to finally obtaining the following expression for P_R:

$$P_R = \frac{(2k-1)(k-1)\theta_{SI} y_R + (k-1)k y_P + k^2 wa}{(2k-1)(k + \theta_{SI}(k-1))}$$

B Exposition of the duopoly case

In the main text, we have limited ourselves to detailing the exposition of the economy in the case we have a monopoly in the (SC) state, i.e. in the case a challenger who innovates finds it optimal to charge a price which will ensure that the highest quality is attractive for both consumer groups. We will now present the main equilibrium equations in the case we have a duopoly in the (SC) state. For the sake of notation brevity, the time and sector subscripts are dropped in this Appendix section; one should simply keep in mind that the variable n is sector-specific, and designates the number of innovations having occurred so far within a *particular* sector. We will also only present the model building blocks which *differ* from the monopoly case; the consumer problem, the pricing problem of firms in the (SI) state as well as the equilibrium on the labor market remain unchanged, and are hence not developed again in this Appendix section.

Prices and profits in the (SC) state

First, note that a further assumption needs to be made so as to ensure that a duopoly can indeed be a possible equilibrium price regime. Indeed, as argued by Zweimuller and Brunner (2005), the pricing problem faced by firms in a given sector being in the (SC) state can be considered as an infinitely repeated game between the quality leader and the producer of the second-best quality. The monopoly pricing strategy as we have described it in the main text is a Nash equilibrium of the stage game. On the other hand, if we assume that both the leader and the follower have positive market shares (i.e. if we want to define a pricing strategy compatible with a duopoly), no pair of prices (p_n, p_{n-1}) represents a Nash equilibrium of the stage game: *given* the price charged by the other, at least one of the two firms always has an incentive to deviate. It is however possible to guarantee the existence of a duopoly equilibrium, under the further condition on the punishment strategies that *no firm is punished if it changes its price without affecting the other firm's profit* (Proof: cf Zweimuller and Brunner (2005), p. 242).

In the case of such an equilibrium, the new quality leader chooses to charge the highest possible price enabling him to capture the group of rich consumers $p^T_{\{n-1,n\}}(R)$, given the expected strategy of the producer of the second-best quality. The former quality leader charges the highest possible price enabling him to capture the poor group of consumers $p^T_{\{n-2,n-1\}}(P)$, given that the producer of quality q_{n-2} engages in marginal cost pricing (i.e. $p_{n-2} = wa$). Those two prices actually correspond to p^P_{SI} and p^R_{SI} as defined by (13) and (14). Hence, when the market structure is a duopoly in the (SC) state, *both types of consumers pay systematically the same price for the consumed quality, whether the industry is in state (SC) or (SI).* We define the profits $\pi_L(n)$ and $\pi_F(n)$ accruing to the producers of the first-best and the second-best qualities in the (SC) state, as well as the profits $\pi^D_{SI}(n)$

of the discriminating monopolist in the (SI) state:[30]

$$\pi_L(n) = (1-\beta)L\left(\frac{k-1}{k}\right)\left[k^{n-1}\frac{c_P}{Q_P^D}+k^n\frac{c_R}{Q_R^D}\right] \tag{31}$$

$$\pi_F(n) = \beta L k^{n-1}\left(\frac{k-1}{k}\right)\frac{c_P}{Q_P^D} \tag{32}$$

$$\pi_{SI}^D(n) = L\left(\frac{k-1}{k}\right)\left[k^{n-1}\frac{c_P}{Q_P^D}+(1-\beta)k^n\frac{c_R}{Q_R^D}\right] \tag{33}$$

with the "quality consumption indices" for both income groups Q_P^D and Q_R^D taking the following form in the case we have a duopoly in the (SC) state:

$$Q_P^D = \int_0^1 k^{n(s)-1}ds; \qquad Q_R^D = \int_0^1 k^{n(s)}ds \tag{34}$$

R&D sector

In the case we have a duopoly in the (SC) state, the main modification is that the value of a "leapfrogged" quality leader $v_F(n+1)$ does not fall to zero: since the new leader abandons the lower part of the market so as to be able to charge a higher price to the rich consumers, the former leader still makes positive profits $\pi_F(n+1)$. We hence now have three Hamilton-Jacobi-Bellman equations. In industries being in the (SC) state, both the incumbent and the former leader face the two following HJB equations:

$$\begin{aligned} rv_{SC}^D(n) &= \max_{\phi_I\geq 0}\Big\{\pi_L(n) - wF\frac{k^{n+1}}{Q}\phi_I \\ &+ \phi_I\big(v_{SI}^D(n+1)-v_{SC}^D(n)\big)+\phi_C\big(v_F(n+1)-v_{SC}^D(n)\big)\Big\} \end{aligned} \tag{35}$$

$$\begin{aligned} rv_F(n) &= \max_{\phi_F\geq 0}\Big\{\pi_F(n) - wF\frac{k^{n+1}}{Q}\phi_F \\ &+ \phi_F\big(v_{SC}^D(n+1)-v_F(n)\big)+(\phi_C+\phi_I)\big(v_C-v_F(n)\big)\Big\} \end{aligned} \tag{36}$$

In industries being in the (SI) state, the incumbent faces the following HJB equation:

$$\begin{aligned} rv_{SI}^D(n) &= \max_{\phi_{DI}\geq 0}\Big\{\pi_{SI}^D(n) - wF\frac{k^{n+1}}{Q}\phi_{DI} \\ &+ \phi_{DI}\big(v_{SI}^D(n+1)-v_{SI}^D(n)\big)+\phi_C\big(v_F(n+1)-v_{SI}^D(n)\big)\Big\} \end{aligned} \tag{37}$$

[30] Even though the pricing strategy of a "fully differentiated" monopolist in the (SI) state does not differ from the monopoly case, we still need to differentiate π_{SI} from π_{SI}^D since, as shown below, the "quality consumption index" of the poor group does not take the same form in both cases.

The three corresponding first-order conditions are of the following form:

$$(-wF\frac{k^{n+1}}{Q} + v^D_{SI}(n+1) - v^D_{SC}(n))\phi_I = 0, \quad \phi_I \geq 0 \tag{38}$$

$$(-wF\frac{k^{n+1}}{Q} + v^D_{SC}(n+1) - v_F(n))\phi_F = 0, \quad \phi_F \geq 0 \tag{39}$$

$$(-wF\frac{k^{n+1}}{Q} + v^D_{SI}(n+1) - v^D_{SI}(n))\phi_{DI} = 0, \quad \phi_{DI} \geq 0 \tag{40}$$

As in the monopoly case, we have that combined with (38), (40) entails either $\phi_{DI} = 0$ or $v^D_{SC}(n) = v^D_{SI}(n)$. The second possibility cannot be true, since $\pi^D_{SI}(n) > \pi_L(n)$: we hence necessarily have that $\phi_{DI} = 0$. Combined with the free-entry condition (17), (39) entails either $\phi_F = 0$ or $v_F(n) = 0$. The second possibility cannot be true, since the follower's profits $\pi_F(n)$ are strictly positive: we hence necessarily have that $\phi_F = 0$. Plugging this value back into (36), we obtain that $v_F(n) = \frac{\pi_F(n)}{\rho+\phi_C+\phi_I}$. Plugging the free-entry condition (17) and the first-order condition (38) in the HJB equations (35) and (37), substituting for the profit values obtained in (31), (32) and (33), and noticing that in the duopoly case we simply have $Q^D_P = (1/k)Q$, it is possible to obtain the 2 following expressions, equating incurred R&D costs and expected profits in both possible states:

$$wF = \frac{(1-\beta)\left(\frac{k-1}{k}\right)L(c_P + c_R) + \phi_C \frac{\beta L(k-1)c_P}{r+\phi_C+\phi_I}}{r+\phi_C} \tag{41}$$

$$\left(\frac{k+1}{k}\right)wF = \frac{\left(\frac{k-1}{k}\right)L\left[(1-\beta)c_R + c_P\right] + \phi_C \frac{\beta L(k-1)c_P}{r+\phi_C+\phi_I}}{r+\phi_C} \tag{42}$$

Definition of the equilibrium and the BGP

Definition 2 *In the case we have a duopoly market structure in the (SC) state, an equilibrium is defined by a time path for consumption of the homogenous good for both types of consumers* $\{c_i(t)\}^\infty_{i=(R,P),t=0}$ *that satisfies* (6)*, a time path for the "quality index" for both types of consumers* $\{Q_i(t)\}^\infty_{i=(R,P),t=0}$ *that satisfies* (7)*, time paths for innovation probabilities (and corresponding sector-specific R&D expenditures) by incumbents and challengers* $\{\phi_C(t), \phi_I(t)\}^\infty_{t=0}$ *that satisfy* (17) *and* (38)*, time paths for sector-specific prices and profits* $\{p^P_{SI}(s,t), p^R_{SI}(s,t), \pi_L\big(n(s,t)\big), \pi_F\big(n(s,t)\big), \pi_{SI}\big(n(s,t)\big)\}^\infty_{s\in(0,1),t=0}$ *that satisfy* (13), (14)*,* (31)*,* (32) *and* (33)*, and a time path of the interest rate* $\{r(t)\}^\infty_{t=0}$ *which satisfies* (8).

Once again, we define a BGP as an equilibrium along which every variable grows at a constant rate, either null or positive. The computation of the values of the shares θ_{SC} and θ_{SI} is exactly similar to the one carried out in the monopoly case. On the other hand, the pricing strategy of a successful *challenger* in the (SC) state leads to economy-wide price indices that differ from the ones obtained in the monopoly case. More precisely, we have $P^D_P = \int_0^1 p^P_{SI} ds$ and $P^D_R = \int_0^1 p^R_{SI} ds$. Using (13) and (14) and keeping in mind that

$Q_P^D = \int_0^1 k^{n(s)-1} ds = (1/k) Q_R^D$, it is straightforward to obtain:

$$P_P^D = \frac{(k-1)y_P + kwa}{2k-1} \tag{43}$$

$$P_R^D = \frac{k-1}{2k-1} y_R + \frac{k(k-1)}{(2k-1)^2} y_P + \frac{k^2 wa}{(2k-1)^2} \tag{44}$$

Using (43) and (44), it is finally possible to obtain the following expression of the homogenous good consumption of both consumer groups c_P^D and c_R^D in the duopoly case:

$$c_P^D = \left(\frac{k}{2k-1}\right)(y_P - wa) \tag{45}$$

$$c_R^D = \frac{k(2k-1)y_R - k(k-1)y_P - k^2 wa}{(2k-1)^2} \tag{46}$$

Proposition 5 (Existence and uniqueness of a steady state equilibrium in the duopoly case): *Under the parametric conditions* $(*)-(**)$ *(cf. Appendix E) and for low enough values of* d *and* F*, there exists a unique BGP along which we necessarily have a duopoly in the (SC) state and in which both incumbents and challengers invest strictly positive amounts in R&D* ϕ_I *and* ϕ_C*.*

Proof: cf. Appendix E. Note that while the existence of a unique positive solution for the BGP as defined in Definition 2 can be proved analytically, the absence of closed-form solutions makes it necessary to resort to numerical simulations so as to define parametric intervals in which the defined equilibrium is robust and unique. □

C Existence and uniqueness of the BGP in the monopoly case

Along the BGP, (8) implies that we have $r = \rho$. Using Proposition 3 (cf Appendix A) as well as (29), we also have $\frac{Q}{Q_P} = \frac{k(\phi_I+\phi_C)}{k\phi_C+\phi_I}$ along the BGP. Using (22) so as to substitute for c_P in (23) and keeping in mind that y_R and y_P are linear in Ω along (3), characterizing the BPG then boils down to solving the following 5 equations for values of ϕ_C, ϕ_I, c_R, c_P and overall wealth Ω:

$$wF(\rho + \phi_C) = L\left(\frac{k-1}{k}\right) c_P \frac{k(\phi_I + \phi_C)}{k\phi_C + \phi_I} \tag{47}$$

$$wF(\rho + \phi_C) = L(1-\beta)\left(\frac{k-1}{k}\right) c_R \tag{48}$$

$$wL = wFk\phi_C + \frac{wFk\phi_I\phi_C}{\phi_I + \phi_C} + awL + L\left[\beta c_P + (1-\beta)c_R\right] \tag{49}$$

$$c_P = \left(\frac{k}{2k-1}\right)(y_P - wa) \tag{50}$$

$$c_R = \frac{k(\phi_I + \phi_C)\big((2k-1)y_R - (k-1)y_P - kwa\big)}{(2k-1)(k\phi_C + (2k-1)\phi_I)} \tag{51}$$

We first notice that since $a < 1$ and $y_R > y_P$, positive values of ϕ_C, ϕ_I and of the overall wealth Ω necessarily entail positive values for c_P and c_R along (50) and (51). Using (47) and (48) so as to substitute for c_P and c_R, (49) then yields the following expression for ϕ_I

as a function of ϕ_C:

$$\phi_I = \phi_C \frac{Fk(\rho(1+\beta) + (k+\beta)\phi_C) - (1-a)(k-1)L}{(1-a)(k-1)L - F(k(\rho - \phi_C) + 2k^2\phi_C + \beta(\rho + \phi_C))} \tag{52}$$

Equating the right-hand sides of (47) and (48) and using (50) and (51) to substitute for c_P and c_R, it is also possible to express Ω as a function of ϕ_C and ϕ_I:

$$\Omega = \frac{w\left(F(\rho+\phi_C)(2k-1)(k\phi_C + \phi_I) - k(k-1)(1-a)L(\phi_I + \phi_C)\right)}{d(k-1)k\rho(\phi_C + \phi_I)} \tag{53}$$

Substituting for Ω and ϕ_I using (52) and (53), it is finally possible to transform (47) into a second-degree polynomial in ϕ_C, which when solved yields:

$$\phi_C = \frac{A \pm \sqrt{B + A^2}}{D}$$

with A, B and D having the following analytical expressions:

$$A = (1-a)(k-1)L\Big((1-d)(k^2-1) + k(2-\beta)\Big) - Fk\rho\Big(4k^2 - 1 + d(k(2+3\beta) + 1 - 2k^2(1+\beta))\Big) \tag{54}$$

$$B = 8Fk^2\rho(2k-1-d(k-1)(1+\beta))\Big((1-a)(k-1)L(d + k(2-\beta) - 1) - Fk\rho(2k + d(1+\beta k) - 1)\Big) \tag{55}$$

$$D = 4Fk^2(2k-1-d(k-1)(1+\beta)) > 0 \tag{56}$$

We now define several expressions, that will prove useful for identifying the conditions ensuring positive values for ϕ_C, ϕ_I and Ω.

$$\begin{aligned}
(a) &= \left(\frac{k}{k-1}\right)(\rho(1+\beta) + \phi_C(k+\beta)) \\
(b) &= (1-a)L/F \\
(c) &= \frac{\rho(k+\beta) + \phi_C(k(2k-1)+\beta)}{k-1} \\
(d) &= (\rho + \phi_C)\left(\frac{2k-1}{k(k-1)}\right)\left(\frac{k\phi_C + \phi_I}{\phi_C + \phi_I}\right) \\
(e) &= \rho\left(\frac{k}{k-1}\right)\frac{2k-1+d(1+\beta k)}{2k-1+(1-\beta)d}
\end{aligned}$$

$(b) > (e)$ is sufficient (but not necessary) so as to ensure that we obtain a unique positive solution for ϕ_C, since it entails $B > 0$, which itself implies $\sqrt{A^2+B} > A$ regardless of the sign of A. We then have $\phi_{C1} = \frac{A - \sqrt{A^2+B}}{D} < 0$, and:

$$\phi_{C2} = \frac{A + \sqrt{A^2+B}}{D} > 0 \tag{57}$$

We can then see from (52) that a positive solution for ϕ_I is obtained provided that we have $\phi_C > 0$ and $(a) > (b) > (c)$. Finally, for positive values of ϕ_C and ϕ_I, the condition $(d) > (b)$ is sufficient for (53) to yield a positive value for the overall wealth Ω. We need to ensure that those different inequalities are compatible. So as to guarantee a unique positive solution for ϕ_C, ϕ_I and Ω, the following conditions are hence sufficient (but not necessary):

$$\begin{aligned}(a) > (b) \quad &> \quad (e) > (c) \qquad (*)\\ (d) \quad &> \quad (b) \qquad\quad (**)\end{aligned}$$

We will now determine the conditions on the parameters of the model for conditions $(*)-(**)$ to be respected.

First, since none of the parameters appearing in (b) are present in (e), **it is always possible to choose values of a, F and L such that** $(b) > (e)$, ensuring $\phi_C > 0$.

We can reformulate $(e) = \rho k\left(1 + \frac{\beta d(1+k)}{2k-1+(1-\beta)d}\right)$. Determining the sign of $(a) > (e)$ is then equivalent to determining the sign of the following 2nd-degree polynomial in k:

$$(a) > (e) \;\Leftrightarrow\; 2\phi_C k^2 + \Big(\rho\beta(2-d) + \phi_C(2\beta - 1 - (1-\beta)d)\Big)k - (\rho + \phi_C)\beta(1-(1-\beta)d) - \rho\beta d > 0$$

Provided we have $\phi_C > 0$ and $\beta > \frac{1+d}{2+d}$, the coefficients of the second- and first-degree terms are positive, ensuring the inequality for k big enough. **We hence have (a)>(e) for sufficiently high values of k and β.** Again, since those two conditions do not put any structure on the parameters present in (b), **it is then always possible to choose values of a, F and L such that** $(a) > (b) > (e)$.

$(e) > (c)$ can be rewritten as $\rho k \frac{\beta(1+d)k}{2k-1-\beta k+d} > \rho\beta + \phi_C(k(2k-1)+\beta)$. Provided we have $\frac{\beta(1+d)k}{2k-1-\beta k+d} > 1$ (which is the case for $\beta > \frac{2k-(1-d)}{k(2+d)}$), $(e) > (c)$ is then guaranteed under the condition $(\rho - 2\phi_C)k^2 + \phi_C k - \rho\beta > 0$. Noticing that $\phi_C \to 0$ when $k \to \infty$,[31] we necessarily have $\phi_C < \rho/2$ for k big enough, which ensures that the condition identified above is respected. **We hence have (e)>(c) for sufficiently high values of k.**

Finally, dividing both the numerator and the denominator of ϕ_I as given in (52) by $\phi_C k^2$, we obtain $\phi_I = \phi_C \frac{E}{G}$ with $E = F + \frac{F\rho(1+\beta)}{k\phi_C} + \frac{F\beta}{k} - \frac{(1-a)(k-1)L}{k^2\phi_C}$ and $G = \frac{(1-a)(k-1)L}{k^2\phi_C} - F\left(2 + \frac{\rho}{k\phi_C} - 1/k + \frac{\beta\rho}{k^2\phi_C} + \frac{\beta}{k^2}\right)$. Noticing that $\lim_{k\to+\infty}(k\phi_C) = +\infty$, we have $\lim_{k\to+\infty}\left(\frac{E}{G}\right) = -(1/2)F$, entailing $\lim_{k\to+\infty}(\phi_I) = 0$. We then notice that $(d) > (b)$ can be rewritten as $(\rho + \phi_C)\left(\frac{2k-1}{k-1}\right)(1 + (1/k)\frac{E}{G}) > (1-a)L/F(1 + \frac{E}{G})$: as $k \to +\infty$, the left-hand side of this inequality tends to $2\rho > 0$, while the right-hand side tends to $(1-a)L/F(-1/2F + 1) < 0$. This entails that **we necessarily have (d)>(b) for sufficiently high values of k.**

The intuitions concerning those conditions on the parameter values are commented in the main text of this paper.

The set of parametric conditions $(*)-(**)$, respected for sufficiently high enough values of k and β, are hence sufficient so as to guarantee that **there exists systematically necessarily one (and no more than one) positive solution** for ϕ_C, ϕ_I and Ω, entailing positive solutions for c_P and c_R respectively given by (50) and (51).

[31] More precisely, we have $\lim_{k\to\infty}\phi_C = \frac{(1-a)(1-d)L - 2F\rho(2-d(1+\beta)) + \sqrt{((1-a)(1-d)L - 2F\rho(2-d(1+\beta)))^2}}{4F(2-(1+\beta)d)}$, which is equal to zero provided we have $4 > (b)$, which is itself guaranteed by $(a) > (b)$ for sufficiently high values of k.

Robustness of the monopoly price regime in the (SC) state

Proving that the system of five equations (47)-(51) admits a unique and positive solution in $(\phi_I, \phi_C, \Omega, c_P, c_R)$ is however not sufficient so as to demonstrate the existence and the uniqueness of the defined BGP. Indeed, we have assumed from the beginning that the equilibrium market structure chosen by the new leader in the (SC) state is a **monopoly**. We now need to check that for the obtained values for c_P, c_R, ϕ_C, ϕ_I and Ω, this specific price regime indeed represents a *robust* equilibrium, i.e. **the new leader does not prefer the alternative regime when comparing expected profits**. More formally, the condition for a monopoly to occur is of the form (we drop the sector dependence for the sake of notation brevity):

$$\underbrace{\pi_{SC}(n) + \phi_I^M \pi_{SI}(n+1)}_{(M)} \geq \underbrace{\pi_L(n) + \phi_I^M \pi_{SI}^D(n+1) + \frac{\phi_C^M}{\rho + \phi_C^M + \phi_I^M}\left(\pi_F(n+1) + \phi_I^M \pi_F(n+2)\right)}_{(D)} \tag{58}$$

with the supercript M referring to the fact we compute the different profits for the values of Ω^M, ϕ_C^M and ϕ_I^M obtained when solving for an equilibrium with a **monopoly** market structure. Expressions for π_M, π_{SI}, π_L, π_F and π_{SI}^D are given by equations (12), (15), (31), (32) and (33). Substituting for those and multiplying both sides of the inequality by $\frac{Q}{k^n L(k-1)}$, we are left to compare (M) and (D) taking the following form:

$$\begin{aligned}
(M) &= c_P\left(\frac{\phi_I + \phi_C}{\phi_I + k\phi_C}\right) + \phi_I\left[c_P\left(\frac{\phi_I + \phi_C}{\phi_I + k\phi_C}\right) + (1-\beta)c_R\right] \\
(D) &= (1-\beta)\left(\frac{c_R^D + c_P}{k}\right) + \phi_I\left[c_P + (1-\beta)c_R^D\right] + \frac{\phi_C(1+k\phi_I)}{\rho + \phi_C + \phi_I}\beta c_P
\end{aligned}$$

with c_P, c_R and c_R^D given by (27), (28) and (46). Intuitively, it is straightforward to conjecture that the monopoly state should have higher chances to dominate for high values of β, since it entails a larger size of the group of "poor" consumers. For β close to 1, (M) and (D) can be approximated in the following way:

$$\begin{aligned}
(M) &= \underbrace{c_P\left(\frac{\phi_I + \phi_C}{\phi_I + k\phi_C}\right)}_{(1)^M} + \underbrace{\phi_I\left[c_P\left(\frac{\phi_I + \phi_C}{\phi_I + k\phi_C}\right) + (1-d)\Omega\rho\left(\frac{k\phi_I(\phi_I + \phi_C)}{k\phi_C + (2k-1)\phi_I}\right)\right]}_{(2)^M} \\
(D) &= \underbrace{(1-d)\Omega\rho\left(\frac{1}{2k-1}\right)}_{(1)^D} + \underbrace{\frac{\phi_C}{\rho + \phi_I + \phi_C}c_P}_{(1')^D} + \underbrace{\phi_I\left[c_P\left(1 + \frac{k\phi_C}{\rho + \phi_I + \phi_C}\right) + (1-d)\Omega\rho\left(\frac{k}{2k-1}\right)\right]}_{(2)^D}
\end{aligned}$$

The terms labelled $(1)^M$ and $(1)^D$ are an approximation of the *immediate* profits accruing to a successful challenger when β gets close to 1.[32] The terms labelled $(1')^D$, $(2)^M$ and $(2)^D$ are subsequent profits, that are only being reaped when a further innovation race

[32] One might be surprised that there are still positive profits in the "duopoly case" when β is close to 1. The term $(1)^D$ expresses the fact that for higher values of β (and provided we keep d fixed), the people belonging to the wealthy group are fewer but richer (since there is a higher concentration of wealth in their hands): hence, there are still positive profits to be reaped from specifically targeting this group in the "duopoly case".

is won (either by the incumbent or by another challenger), and that are hence weighted by the innovation probabilities ϕ_C and/or ϕ_I. For high values of k, those terms become negligible in regard of the immediate, first-round profits $(1)^M$ and $(1)^D$. Demonstrating the robustness of a "monopoly" equilibrium is then equivalent to proving $(1)^M > (1)^D$. Using (27) so as to substitute for c_P, this inequality can be reformulated in the following way:

$$(1)^M > (1)^D \;\Leftrightarrow\; d\left(\frac{1}{2k-1}\right)\Omega\rho\underbrace{\left(\frac{k(\phi_I+\phi_C)}{k\phi_C+\phi_I}\right)}_{>1}+\frac{wk(1-a)}{2k-1} > (1-d)\left(\frac{1}{2k-1}\right)\Omega\rho$$

It is clear from the expression above that for high enough values of β (which were already requested so as to guarantee conditions (*) and (**)), the condition $d > 1/2$ (***) is sufficient so as to guarantee $(1)^M > (1)^D$, ensuring the robustness of the "monopoly" equilibrium.

Hence, under the parametric conditions (*)-(***), the positive equilibrium described by $(47)-(51)$ **defines a unique BGP where the market structure in the (SC) case is necessarily a monopoly.** This ends the proof. $\square$

D Demonstration of the comparative statics

Comparative statics in the case of a variation in d

We first consider the variation of $\phi_C = \frac{A+\sqrt{B+A^2}}{D}$ (the values of A, B and D being defined by (54), (55) and (56) in Appendix C) following a shock on d:

$$\frac{\partial \phi_C}{\partial d} = \left(\underbrace{\left(1+\frac{A}{\sqrt{\Delta}}\right)\frac{\partial A}{\partial d}}_{T_1}+\underbrace{\frac{\partial B}{\partial d}\frac{1}{2\sqrt{\Delta}}}_{T_2}\right)D-\underbrace{(A+\sqrt{\Delta})\frac{\partial D}{\partial d}}_{T_3} \tag{59}$$

with $\Delta = B + A^2$, and the following forms for $\frac{\partial A}{\partial d}$, $\frac{\partial B}{\partial d}$ and $\frac{\partial D}{\partial d}$:

$$\frac{\partial A}{\partial d} = Fk\rho\left(2k^2(1+\beta)-1-k(2+3\beta)\right)-(1-a)(k-1)^2(k+1)L \tag{60}$$

$$\begin{aligned}\frac{\partial B}{\partial d} = {} & 8Fk^2\rho\Big[\underbrace{(2k-1-d(k-1)(1+\beta))((1-a)(k+1)L-Fk\rho(1+\beta k))}_{B_{d1}} \\ & \underbrace{-(k-1)(1+\beta)\Big((1-a)(k-1)L(d+k(2-\beta)-1)-Fk\rho(2k+d(1+\beta k)-1)\Big)}_{B_{d2}}\Big]\end{aligned} \tag{61}$$

$$\frac{\partial D}{\partial d} = -4dFk^2(k-1)(1+\beta) < 0 \tag{62}$$

We will now show that under the parametric conditions guaranteeing the existence and uniqueness of the BGP (cf Appendix C), since we have $D > 0$, the sign of (59) is ultimately determined by the sign of $\frac{\partial B}{\partial d}$. So as to proceed to this demonstration, we need to define the following expression:

$$
\begin{aligned}
(f) &= \rho\left(\frac{k}{k-1}\right)\frac{2(2-d(1+\beta))k^2+d(2+3\beta)k-1+d}{(1-d)(k^2-1)+k(2-\beta)}\\
(g) &= \rho\left(\frac{k}{k-1}\right)(1+\beta k)
\end{aligned}
$$

As it can be seen considering (54), the sign of A (that we did not need to establish so as to prove that $\phi_C > 0$ as long as $B > 0$) is given by the sign of $(b)-(f)$. As $k \to +\infty$, we have that $(f) \to \frac{2(2-d(1+\beta))}{1-d}\rho > \frac{2(2-2d)}{1-d}\rho = 4\rho$. On the other hand, we had established that $(d) > (b)$ (cf condition (**) in Appendix C), with $(d) < (\rho+\phi_C)\frac{2k-1}{k-1}$. Since $\lim_{k\to+\infty}(\rho+\phi_C)\frac{2k-1}{k-1} = 2\rho$, we necessarily have that $(f) > (d) > (b)$ for high enough values of k, which entails $A < 0$. Considering (54) and (55), one can furthermore see that B is a polynomial of degree 5 in k, while A is a polynomial of degree 3: for high enough values of k, we have that B becomes negligible in $\sqrt{B+A^2}$, which can be approximated by $\sqrt{A^2} = |A|$. We hence have that as $k \to +\infty$, $\frac{A}{\sqrt{\Delta}} \to 1^-$ and $A+\sqrt{\Delta} \to 0^+$. As a consequence, the terms T_1 and T_3 become negligible, and the sign of $\frac{\partial \phi_C}{\partial d}$ is determined by the sign of T_2, i.e. of $\frac{\partial B}{\partial d}$.

We have $2k-1-d(k-1)(1+\beta) < 0$, and for k sufficiently high we necessarily have $(g) > (b)$, which ensures that the term B_{d1} of (61) is negative. We also have $B_{d2} < 0$ since $(b) > (e)$ (cf condition (*) in Appendix C). As a consequence, we have $\frac{\partial B}{\partial d} < 0$, which entails $\frac{\partial \phi_C}{\partial d} < 0$: **under the conditions ensuring the existence and uniqueness of the BGP, ϕ_C is decreasing along d.**

We then move to determining the variations of ϕ_I following a shock on d:

$$
\frac{\mathrm{d}\phi_I}{\mathrm{d}d} = \frac{\partial \phi_I}{\partial d} + \frac{\partial \phi_I}{\partial \phi_C}\frac{\partial \phi_C}{\partial d}
$$

We first notice that $\frac{\partial \phi_I}{\partial d} = 0$. Using (52), we then reformulate $\phi_I = \phi_C\frac{H}{I}$ with $H = F(k-1)((a)-(b)) > 0$ and $I = F(k-1)((b)-(c)) > 0$. Since we have $\frac{\partial H}{\partial \phi_C} = Fk(K+\beta)$ and $\frac{\partial I}{\partial \phi_C} = -F(k(2k-1)+\beta)$, we obtain the following expression for $\frac{\partial \phi_I}{\partial \phi_C}$:

$$
\frac{\partial \phi_I}{\partial \phi_C} = \frac{HI + \phi_C IFk(k+\beta) + \phi_C HF(k(2k-1)+\beta)}{I^2} > 0 \tag{63}
$$

(63), along with our above result that $\frac{\partial \phi_C}{\partial d} < 0$, yields $\frac{\mathrm{d}\phi_I}{\mathrm{d}d} < 0$: **under the conditions ensuring the existence and uniqueness of the BGP, ϕ_I is decreasing along d.**

Finally, the variation of θ_{SI} can be determined considering the equilibrium condition (47) (cf. Appendix C), which is a reformulation of the R&D free-entry condition (22) and in which $Q/Q_P = \frac{k}{k\theta_{SC}+\theta_{SI}}$ appears in the right-hand side (RHS). Using $\theta_{SC}+\theta_{SI} = 1$, we have $Q/Q_P = \frac{k}{k-(k-1)\theta_{SI}}$, and obtain that this ratio is increasing along θ_{SI}. Following an increase in d, c_P increases and ϕ_C decreases, moving the RHS and the LHS of (47) in two opposite directions. So as to re-establish the equality, Q/Q_P needs to decrease: we necessarily have that θ_{SI} has decreased following the positive shock on d.

Comparative statics in the case of a variation in β

Similarly, we first consider the variation of ϕ_C following a shock on β:

$$\frac{\partial\phi_C}{\partial\beta} = \left(\left(1+\frac{A}{\sqrt{\Delta}}\right)\frac{\partial A}{\partial\beta} + \frac{\partial B}{\partial\beta}\frac{1}{2\sqrt{\Delta}}\right) D - (A+\sqrt{\Delta})\frac{\partial D}{\partial\beta} \tag{64}$$

with the following forms for $\frac{\partial A}{\partial\beta}$, $\frac{\partial B}{\partial\beta}$ and $\frac{\partial D}{\partial\beta}$:

$$\frac{\partial A}{\partial\beta} = k(dFk(2k-3)\rho - (1-a)(k-1)L) \tag{65}$$

$$\begin{aligned}\frac{\partial B}{\partial\beta} = 8Fk^2\rho\Big[&\underbrace{-k(2k-1-d(k-1)(1+\beta))((1-a)(k-1)L+dFk\rho)}_{B_{\beta 1}}\\ &\underbrace{-d(k-1)\Big((1-a)(k-1)L(d+k(2-\beta)-1) - Fk\rho(2k+d(1+\beta k)-1)\Big)}_{B_{\beta 2}}\Big]\end{aligned} \tag{66}$$

$$\frac{\partial D}{\partial\beta} = -4dF(k-1)k^2 < 0 \tag{67}$$

The reasoning we presented when analyzing $\frac{\partial\phi_C}{\partial d}$ still holds here: under the parametric conditions guaranteeing the existence and uniqueness of the BGP, the sign of (64) is ultimately determined by the sign of $\frac{\partial B}{\partial\beta}$. We unequivocally have $B_{\beta 1} < 0$, and since $(b) > (e)$ we have $B_{\beta 2} < 0$. As a consequence, we have $\frac{\partial B}{\partial\beta} < 0$, which entails $\frac{\partial\phi_C}{\partial\beta} < 0$: **under the conditions ensuring the existence and uniqueness of the BGP, ϕ_C is decreasing along β.**

We then move to determining the variations of ϕ_I following a shock on β:

$$\frac{\mathrm{d}\phi_I}{\mathrm{d}\beta} = \frac{\partial\phi_I}{\partial\beta} + \frac{\partial\phi_I}{\partial\phi_C}\frac{\partial\phi_C}{\partial\beta}$$

We have already established that $\frac{\partial\phi_C}{\partial\beta} < 0$ and $\frac{\partial\phi_I}{\partial\phi_C} > 0$. We finally have the following partial derivative of ϕ_I w.r.t. β:

$$\frac{\partial\phi_I}{\partial\beta} = \frac{F(k-1)\phi_C(\rho+\phi_C)((1-a)(k-1)L - Fk(\rho+2k\phi_C))}{\Big(F(k(\rho-\phi_C)+2k^2\phi_C+\beta(\rho+\phi_C)) - (1-a)(k-1)L\Big)^2} \tag{68}$$

Since $\lim_{k\to+\infty}(k\phi_C) = +\infty$, for k high enough we necessarily have $\frac{k}{k-1}(\rho+2k\phi_C) > (b)$, which entails $\frac{\partial B}{\partial\beta} < 0$: **under the conditions ensuring the existence and uniqueness of the BGP, ϕ_I is decreasing along β.**

Finally, the variation of θ_{SI} can again be determined considering the equilibrium condition (47) (cf. Appendix C). Following a positive shock on β, the LHS decreases through the decrease in ϕ_C. So as to re-establish the equality, the RHS needs to decrease identically, and we necessarily have that θ_{SI} has decreased following the shock on β. This ends the proofs. □

E Existence of the BGP in the duopoly case

Along the BGP, (8) implies that we have $r = \rho$. Keeping in mind that y_R and y_P are linear in Ω along (3), characterizing the BPG then boils down to solving the following 5 equations for values of ϕ_C, ϕ_I, c_R^D, c_P^D and overall wealth Ω:

$$
\begin{aligned}
wF(\rho+\phi_C) &= (1-\beta)\left(\frac{k-1}{k}\right)L(c_P+c_R)+\phi_C\frac{\beta L(k-1)c_P}{\rho+\phi_C+\phi_I} && (69)\\
\left(\frac{k+1}{k}\right)wF(\rho+\phi_C) &= \left(\frac{k-1}{k}\right)L\left[(1-\beta)c_R+c_P\right]+\phi_C\frac{\beta L(k-1)c_P}{\rho+\phi_C+\phi_I} && (70)\\
wL &= wFk\phi_C+\frac{wFk\phi_I\phi_C}{\phi_I+\phi_C}+awL+L\left[\beta c_P+(1-\beta)c_R\right] && (71)\\
c_P^D &= \left(\frac{k}{2k-1}\right)(y_P-wa) && (72)\\
c_R^D &= \frac{k(2k-1)y_R-k(k-1)y_P-k^2wa}{(2k-1)^2} && (73)
\end{aligned}
$$

We first notice that since $a<1$ and $y_R>y_P$, positive values of ϕ_C, ϕ_I and of the overall wealth Ω necessarily entail positive values for c_P^D and c_R^D along (72) and (73). Substracting (69) from (70) and using the obtained equation to solve for Ω yields the following expression:

$$\Omega = w\frac{F(2k-1)(\rho+\phi_C)-(1-a)(k-1)Lk\beta}{d(k-1)k\rho\beta} \tag{74}$$

Proceeding to the manipulation (69) $-\left(\frac{k}{k+1}\right)$(70), we obtain the following equilibrium condition:

$$(A_D(\rho+\phi_C)-B_D)(\rho+\phi_C+\phi_I)+\left(\frac{F}{(k-1)L}\right)\phi_C(\rho+\phi_C)=0 \tag{75}$$

with A_D and B_D being defined the following way:

$$
\begin{aligned}
A_D &= \frac{F(-2dk^2\beta+k(2-d(2\beta-1))-1+d\beta)}{d(k-1)k(2k-1)L\beta}\\
B_D &= \frac{(1-a)(1-d)}{d(2k-1)}>0 && (76)
\end{aligned}
$$

Substituting for Ω using (74), it is then possible to express ϕ_I as a function of ϕ_C using (75). More precisely, it is possible to obtain $\phi_I=\psi_R(\phi_C)$, with ψ_R being implicitly defined by $R(\phi_I,\phi_C)=0$:

$$R(.) = \left(\frac{F}{(k-1)L}+A_D\right)\phi_C^2+\left(\left(2A_D+\frac{F}{(k-1)L}\right)\rho-B_D\right)\phi_C+A_D\phi_C\phi_I+(A_D\rho-B_D)\phi_I+(A_D\rho-B_D)\rho$$

Proceeding to substitute for Ω in (71), it is also possible to obtain $\phi_I=\psi_L(\phi_C)$, with ψ_L being implicitly defined by $L(\phi_I,\phi_C)=0$:

$$L(.)=-(Fk+C_D)\phi_C^2+((1-a)L-C_D\rho+D_D)\phi_C-(2Fk+C_D)\phi_I\phi_C+((1-a)L-C_D\rho+D_D)\phi_I$$

with C_D and D_D being defined the following way:

$$
\begin{aligned}
C_D &= \frac{F(2k-1-d(k-1)(1-\beta))}{d(k-1)(2k-1)\beta}>0\\
D_D &= \frac{(1-a)(1-d)kL}{d(2k-1)}>0 && (77)
\end{aligned}
$$

We now proceed to characterizing the curves RR and LL (respectively representing the two functions ψ_R and ψ_L in the (ϕ_C, ϕ_I) plane). More precisely, we will show that 2 sufficient conditions for the two curves to intersect once (and only once) in the upper-right quadrant, i.e. for positive values of ϕ_C and ϕ_I, are the following:

$$\begin{aligned} 0 &< \frac{(1-a)L - C_D\rho + D_D}{2Fk + C} < \frac{B-\rho A}{A} \quad (*) \\ 0 &< B - \left(2A + \frac{F}{(k-1)L}\right)\rho \quad (**) \end{aligned}$$

While we will go back to characterizing and commenting the conditions on the parameters of the model that guarantee $(*) - (**)$ once we have proceeded with the demonstration, note for now that $(*)$ necessarily entails $A_D > 0$.

We have the following expressions for ψ_R and ψ_L:

$$\psi_R(\phi_C) = \frac{\left(\frac{F}{(k-1)L} + A_D\right)\phi_C^2 + \left(\left(2A_D + \frac{F}{(k-1)L}\right)\rho - B_D\right)\phi_C + (A_D\rho - B_D)}{A_D(\rho + \phi_C) - B_D} \quad (78)$$

$$\psi_L(\phi_C) = \frac{(Fk + C_D)\phi_C^2 - ((1-a)L - C_D\rho + D_D)\phi_C}{-(2Fk + C_D)\phi_C + ((1-a)L - C_D\rho + D_D)\phi_I} \quad (79)$$

We first consider the intercepts of RR and LL with the vertical axis: $\psi_R(0) = -\rho$, and $\psi_L(0) = 0$. Considering (78), one can notice that RR displays an asymptote at $\phi_{CM} = \frac{B-\rho A}{A} > 0$ under $(*)$. Similarly, considering (79), one can notice that LL displays an asymptote at $\phi_{Cm} = \frac{(1-a)L - C_D\rho + D_D}{2Fk+C} > 0$ under $(*)$. Finally, since $-\frac{Fk+C_D}{2Fk+C_D} < 0$ and $-1 - \frac{F}{A_D(k-1)L} < 0$ under condition $(*)$, we have $\lim_{\phi_C \to -\infty}(\psi_L) = +\infty$, $\lim_{\phi_C \to -\infty}(\psi_R) = +\infty$, $\lim_{\phi_C \to +\infty}(\psi_L) = -\infty$ and $\lim_{\phi_C \to +\infty}(\psi_R) = -\infty$. We then move to considering the slopes of RR and LL. Using implicit differentiation, we have $\frac{\partial \psi_R}{\partial \phi_C} = -\frac{\partial R/\partial \phi_C}{\partial R/\partial \phi_I}$ and $\frac{\partial \psi_L}{\partial \phi_C} = -\frac{\partial L/\partial \phi_C}{\partial L/\partial \phi_I}$:

$$\frac{\partial R}{\partial \phi_C} = 2\left(A_D + \frac{F}{(k-1)L}\right)\phi_C + \left(\left(2A_D + \frac{F}{(k-1)L}\right) - B_D\right) + A_D\phi_I \quad (80)$$

$$\frac{\partial R}{\partial \phi_I} = (A_D(\rho + \phi_C) - B_D) \quad (81)$$

$$\frac{\partial L}{\partial \phi_C} = -2(Fk + C_D)\phi_C + ((1-a)L - C_D\rho + D_D) - (2Fk + C_D)\phi_I \quad (82)$$

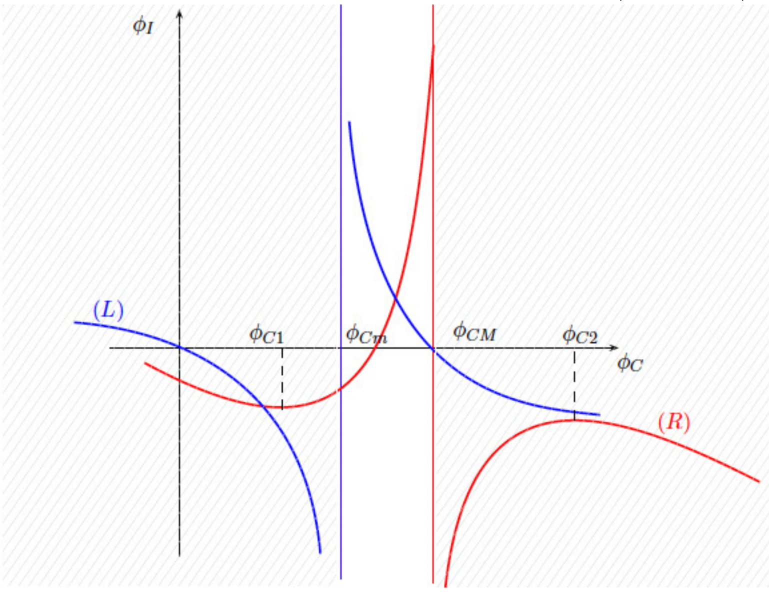
$$\frac{\partial L}{\partial \phi_I} = -(2Fk + C_D)\phi_C + ((1-a)L - C_D + D_D) \quad (83)$$

We first consider the slope of RR. For $\phi_C < \phi_{CM}$, we have $\frac{\partial R}{\partial \phi_I} < 0$, which entails that the sign of $\frac{\partial \psi_R}{\partial \phi_C}$ will be the same than the sign of $\frac{\partial R}{\partial \phi_C}$. The opposite reasoning applies for $\phi_C > \phi_{CM}$. Substituting for ϕ_I using (78), one can finally see that solving for $\frac{\partial R}{\partial \phi_C} = 0$ is equivalent to solving the following 2nd-degree polynomial in ϕ_C:

$$\underbrace{-A_D\left(A_D + \frac{F}{(k-1)L}\right)}_{<0 \text{ under } (*)}\phi_C^2 + \underbrace{2(B_D - A_D\rho)\left(A_D + \frac{F}{(k-1)L}\right)}_{>0 \text{ under } (*)}\phi_C - \underbrace{(B_D - A_D\rho)\left(\rho - (2A_D + \frac{F}{(k-1)L})\rho + B_D\right)}_{>0 \text{ under } (**)} = 0$$

Given the signs of the different terms, this polynomial necessarily admits two positive roots

(ϕ_{C1}, ϕ_{C2}), which itself entails that RR displays two inflexion points: we have $\frac{\partial \psi_R}{\partial \phi_C} < 0$ for $\phi_C \in [0; \phi_{C1}]$ and $\phi_C > \phi_{C2}$, while we have $\frac{\partial \psi_R}{\partial \phi_C} > 0$ for $\phi_C \in [\phi_{C1}, \phi_{C2}]$. Using this information along the existence of an asymptote, we necessarily have that under $(*)-(**)$, RR decreases until ϕ_{C1}, then starts increasing and asymptotically tends to $+\infty$ as $\phi_C \to \phi_{CM}^-$. On the right of the asymptote, RR increases until ϕ_{C2}, then starts decreasing and tends to $-\infty$ as $\phi_C \to +\infty$. Furthermore, going back to the equilibrium condition (75), one can see that we necessarily need $A_D(\rho+\phi_C) - B_D < 0$ for this condition to be met for positive values of *both* ϕ_I and ϕ_C: in other words, RR **only goes above the horizontal axis for values of** $\phi_C < \frac{B-A\rho}{\rho}$, and we necessarily have $\psi_R(\phi_{C2}) < 0$. The corresponding graphical representation of RR can be found below (red curve).



We now move to considering the slope of LL. For $\phi_C < \phi_{Cm}$, we have $\frac{\partial L}{\partial \phi_I} > 0$, which entails that the sign of $\frac{\partial \psi_L}{\partial \phi_C}$ will be the opposite of the sign of $\frac{\partial L}{\partial \phi_C}$. The opposite reasoning applies for $\phi_C > \phi_{Cm}$. We also have that there is no real solution to $\frac{\partial L}{\partial \phi_C} = 0$, which means that LL is either strictly increasing or strictly decreasing along ϕ_C. Since we have $\lim_{\phi_C \to -\infty}(\psi_L) = +\infty$ and $\lim_{\phi_C \to +\infty}(\psi_L) = -\infty$, we necessarily have that LL is strictly decreasing. Considering all the gathered information, we necessarily have that LL intersects the origin point $(0, 0)$, then tends to $-\infty$ as $\phi_C \to \phi_{Cm}^-$. On the right of the asymptote, RR tends to $+\infty$ as $\phi_C \to \phi_{Cm}^+$, and decreases to $-\infty$ as $\phi_C \to +\infty$. In other words, LL **goes above the horizontal axis for values of** $\phi_C > \frac{(1-a)L - C_D\rho + D_D}{2Fk+C}$. The corresponding graphical representation of LL can be found above (blue curve).

Under the conditions $(*)-(**)$, which are respected for low enough values of k and not too low values of β, we hence have that RR and LL necessarily intersect once (and only once) in the upper-right quadrant, i.e. for positive values of ϕ_C and ϕ_I. This intersection necessarily happens for $\frac{(1-a)L - C_D\rho + D_D}{2Fk+C} < \phi_C < \frac{B-\rho A}{A}$. Conditions $(*)-(**)$ hence ensure the existence of a unique positive equilibrium in the duopoly case.

Robustness of the duopoly price regime in the (SC) state

We have hence proved that under conditions $(*) - (**)$, there exists a unique and positive solution to the system of equations defining a BGP with a duopoly price regime in the (SC) state. As already extensively commented in Appendix C, we however still need to make sure that the obtained values for ϕ_I^D, ϕ_C^D, Ω^D, c_R^D and c_P^D indeed make it optimal for the successful challenger to charge a price p_R^D capturing only the rich. In other terms, we need the following condition to be respected:

$$\pi_{SC}(n)+\phi_I^D\pi_{SI}(n+1)\leq\pi_L(n)+\phi_I^D\pi_{SI}^D(n+1)+\frac{\phi_C^D}{\rho+\phi_C^D+\phi_I^D}\left(\pi_F(n+1)+\phi_I^D\pi_F(n+2)\right)\quad(84)$$

with the supercript D referring to the fact we compute the different profits for the values of Ω^D, ϕ_C^D and ϕ_I^D obtained when solving for an equilibrium with a **duopoly** market structure. The condition above is the exact contrary of Condition (58), which necessarily holds in the case of ϕ_C^M, ϕ_I^M and Ω^M for high enough values of β, k and low enough values of d (cf. Appendix C). Fixing low enough values of β and high enough values of d would hence probably imply that Condition (84) is respected, ensuring the existence and the uniqueness of the BGP with a duopoly price regime. However, conditions $(*) - (**)$ hold only for high enough values of β. It is hence a priori not obvious that there exists parameter values for which the duopoly case is robust, and it will depend on the obtained values for ϕ_C^D and ϕ_I^D. The absence of closed-form solutions however makes it necessary to resort to simulations so as to determine whether (84) holds for some parameter values.

Carrying out some simulations for a wide array of parametric values, the following numerical finding emerges:

Numerical finding: *Under the parametric conditions $(*) - (**)$ and for low enough values of F and d, there exists a unique and robust equilibrium BGP in which we have a duopoly in the (SC) state, and the incumbents invest a positive amount in R&D ϕ_I.*

It however appears that the parametric constellations under which the duopoly case is robust and unique are much narrower than their counterpart ensuring a unique and robust BGP with a monopoly in the (SC) state. This also justifies the fact that we focus on the monopoly case in the main text of the paper.